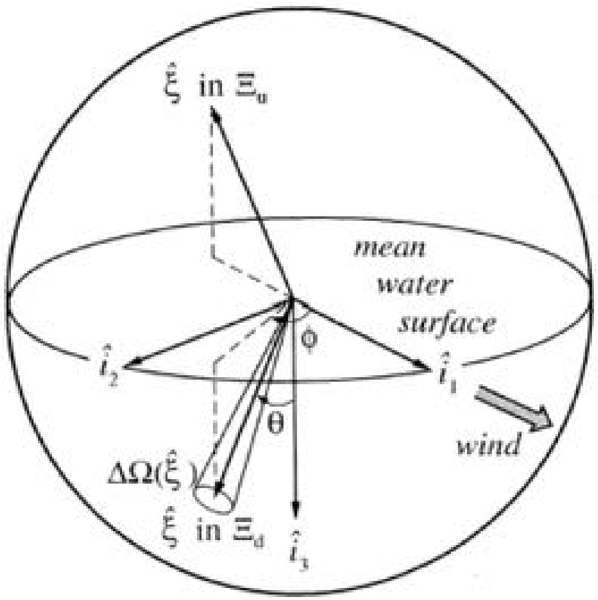
Introduction to Radiative Transfer Modelling



$$\cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} = \ldots$$

HYDROLIGHT

John Hedley, IOCCG Summer Lecture Series, 2026

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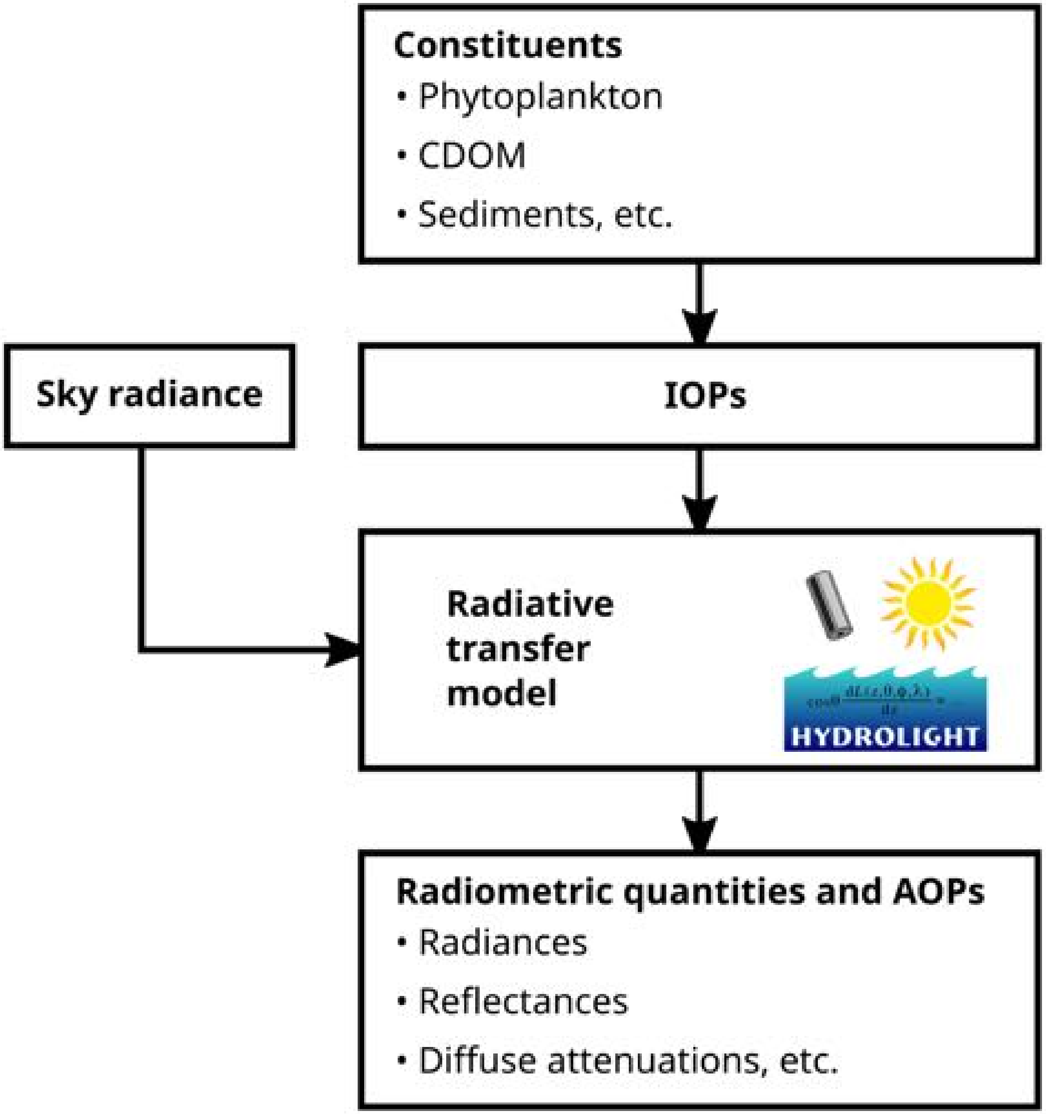
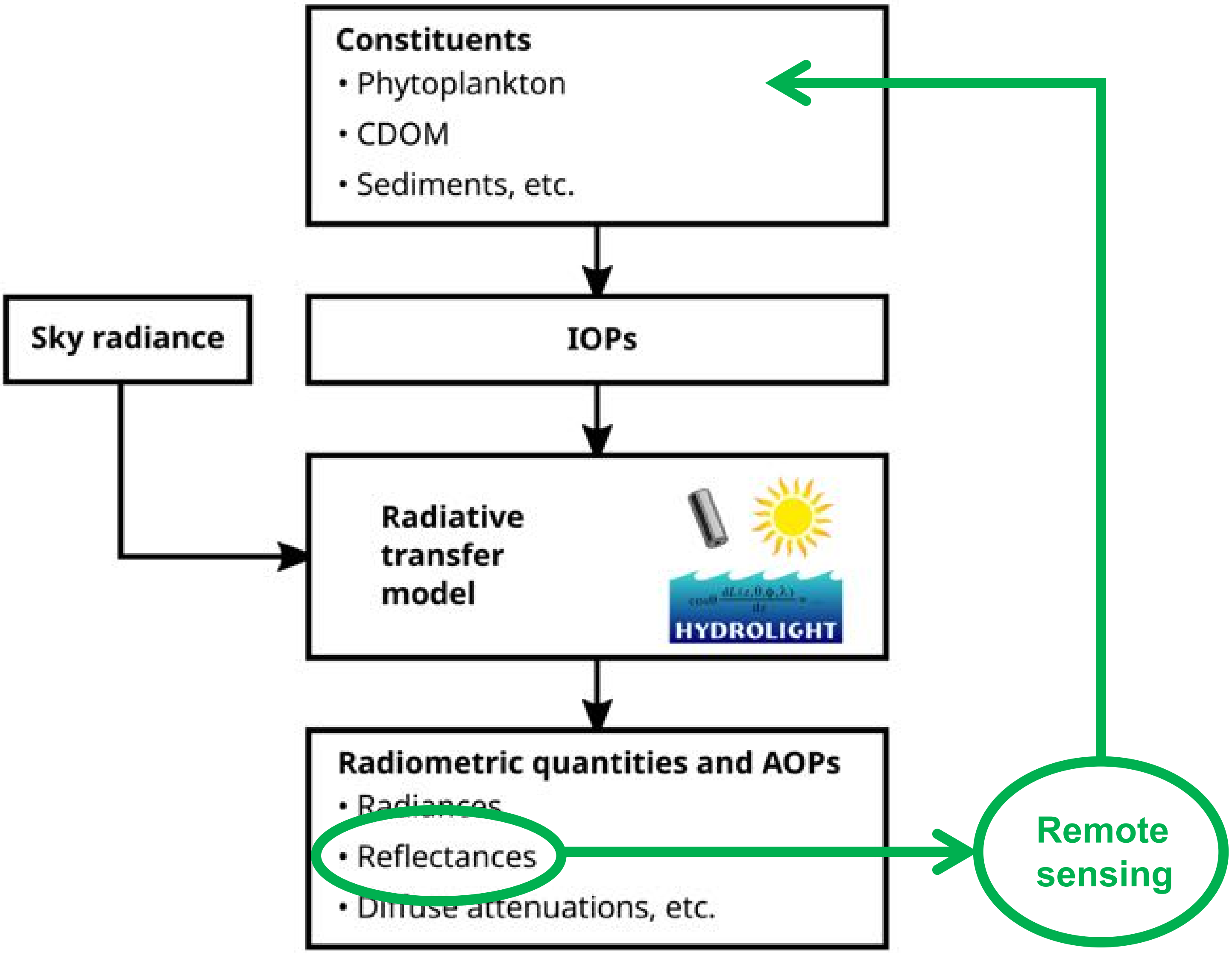
Forward Modelling



Forward Modelling



The role of radiative transfer modelling

Understanding how IOPs relate to AOPs

- Characterise relationship between constituents and reflectance

Feasibility studies

- What might be detectable in the reflectance?
- Sensitivity analyses
- Consequences of variability in constituents on $R_{rs}(\lambda)$

Develop inversion algorithms

- Look-up tables
- Components of semi-analytical algorithms, e.g. convert $r_{rs}(\lambda)$ to $R_{rs}(\lambda)$
- Train machine learning

Learning tool

- Build an intuitive understanding of Hydrological optics

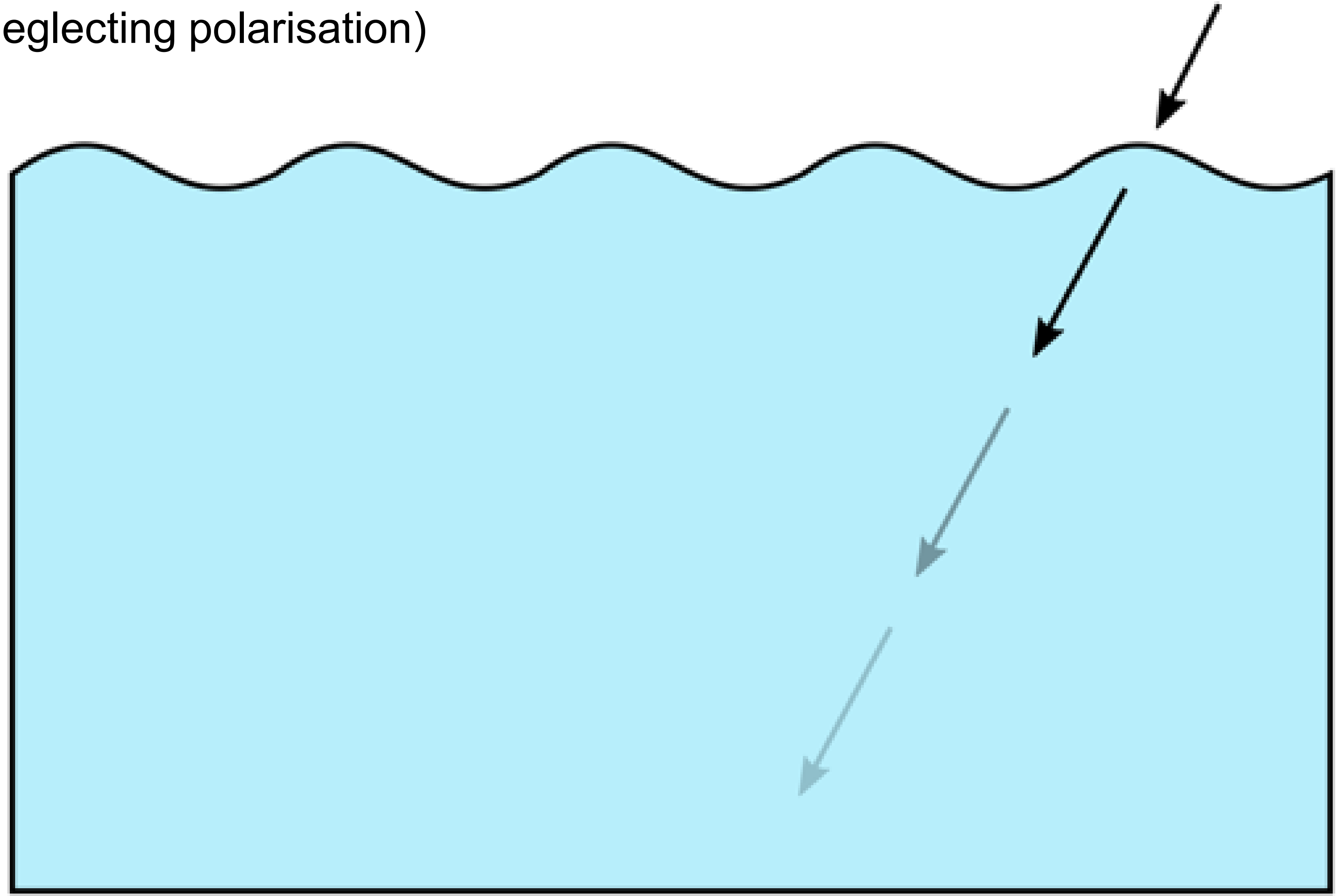
HydroLight

- Developed by Curt Mobley working with Rudy Preisendorfer, starting in 1978.
- Commercial product on PC since 1998.
- Several hundred users in 30+ countries, features in many publications.
- Google scholar search for “HydroLight” returns > 2600 results.
- As of 2017 ownership of HydroLight passed to me (John Hedley) and is now a product of Numerical Optics Ltd.
- Commercial basis facilitates support, maintenance and development of the software.
- Latest version is version 6.0, now available for macOS and Linux.
- Next version 7.0 – in development, currently is in ‘beta test’, adds polarisation and can be configured as a coupled atmosphere-ocean model.

See the document *HydroLight_History.pdf* for more historical info.

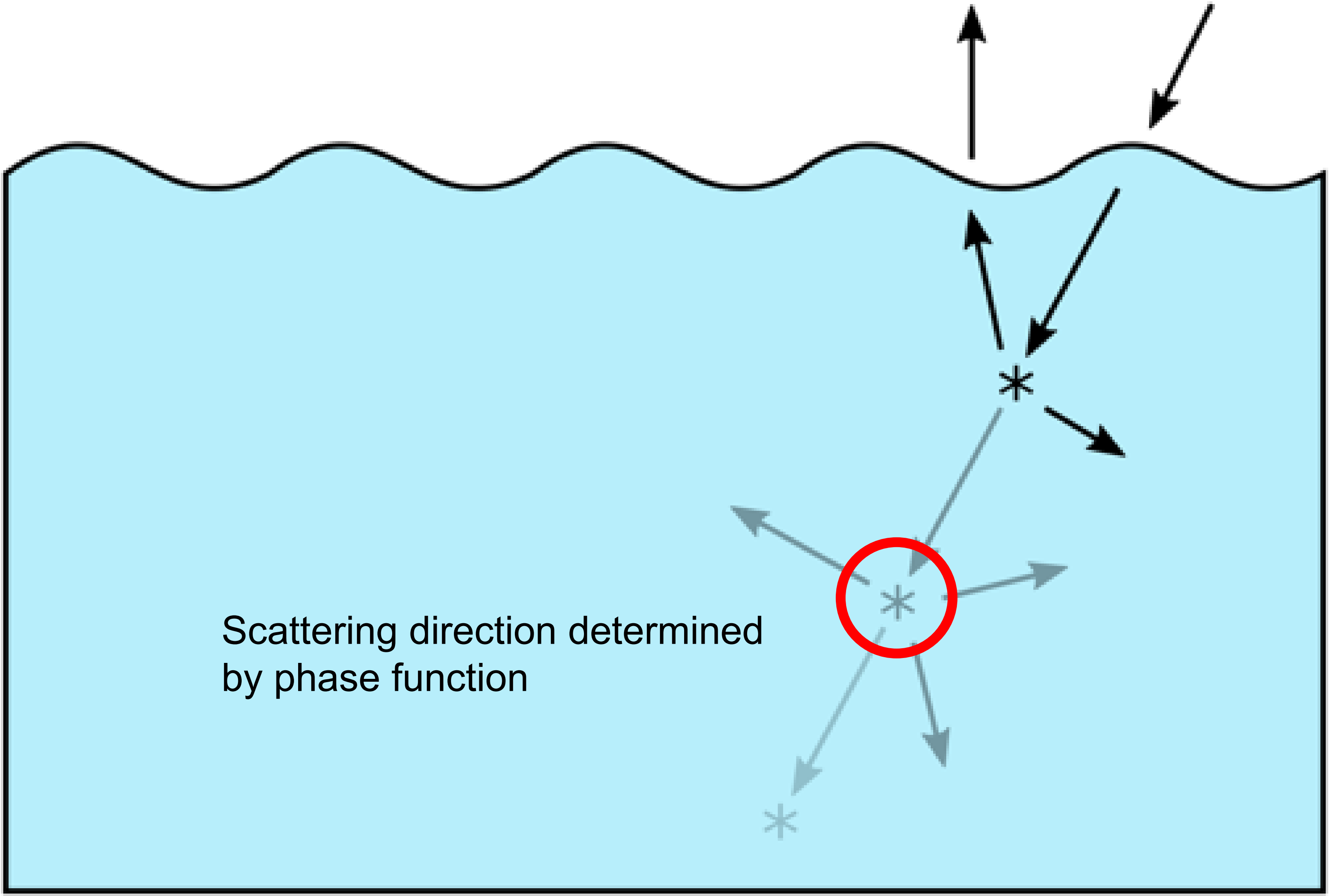
The problem to be solved

(neglecting polarisation)



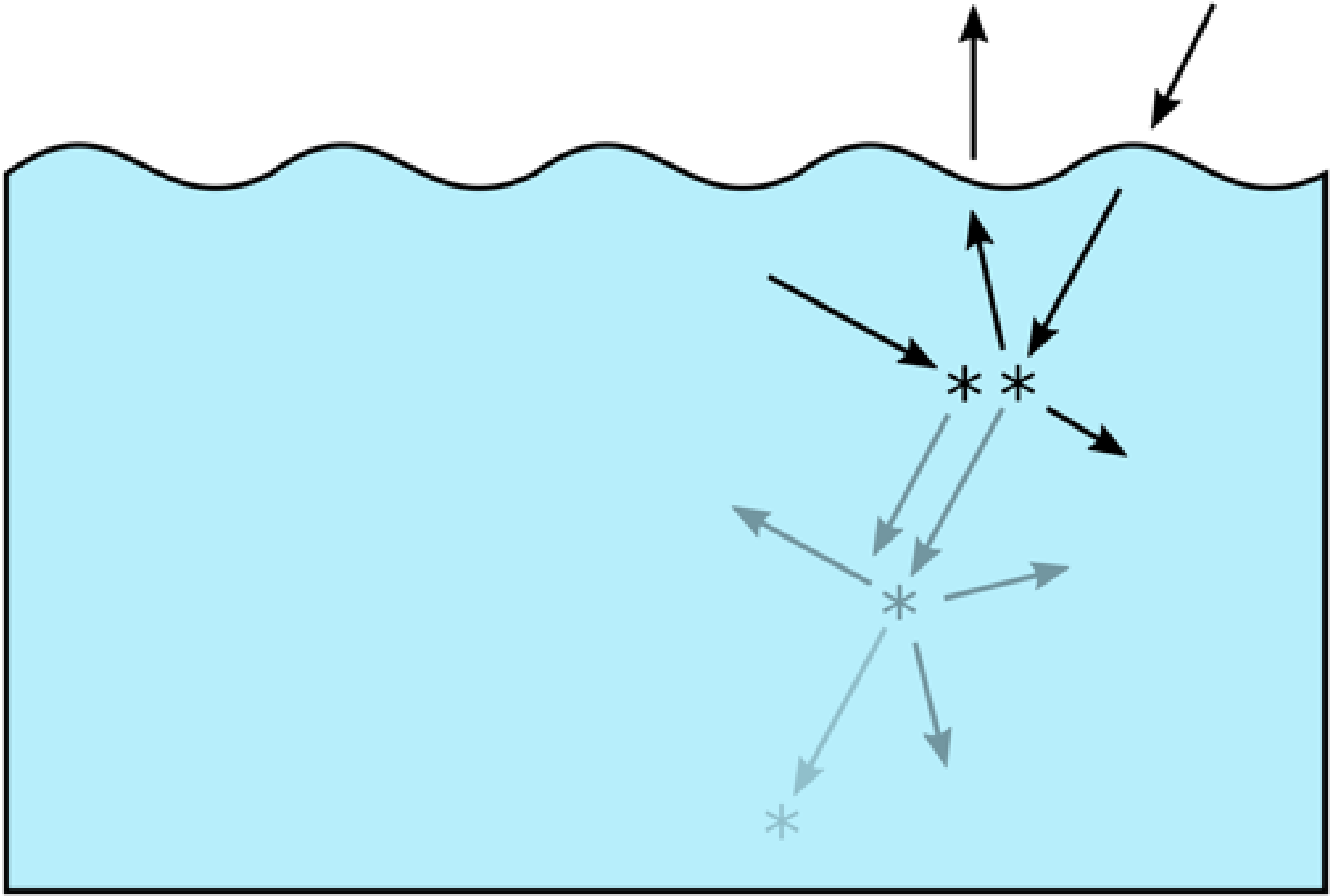
Example 1: No scattering, only absorption

Single scattering - losses to a beam



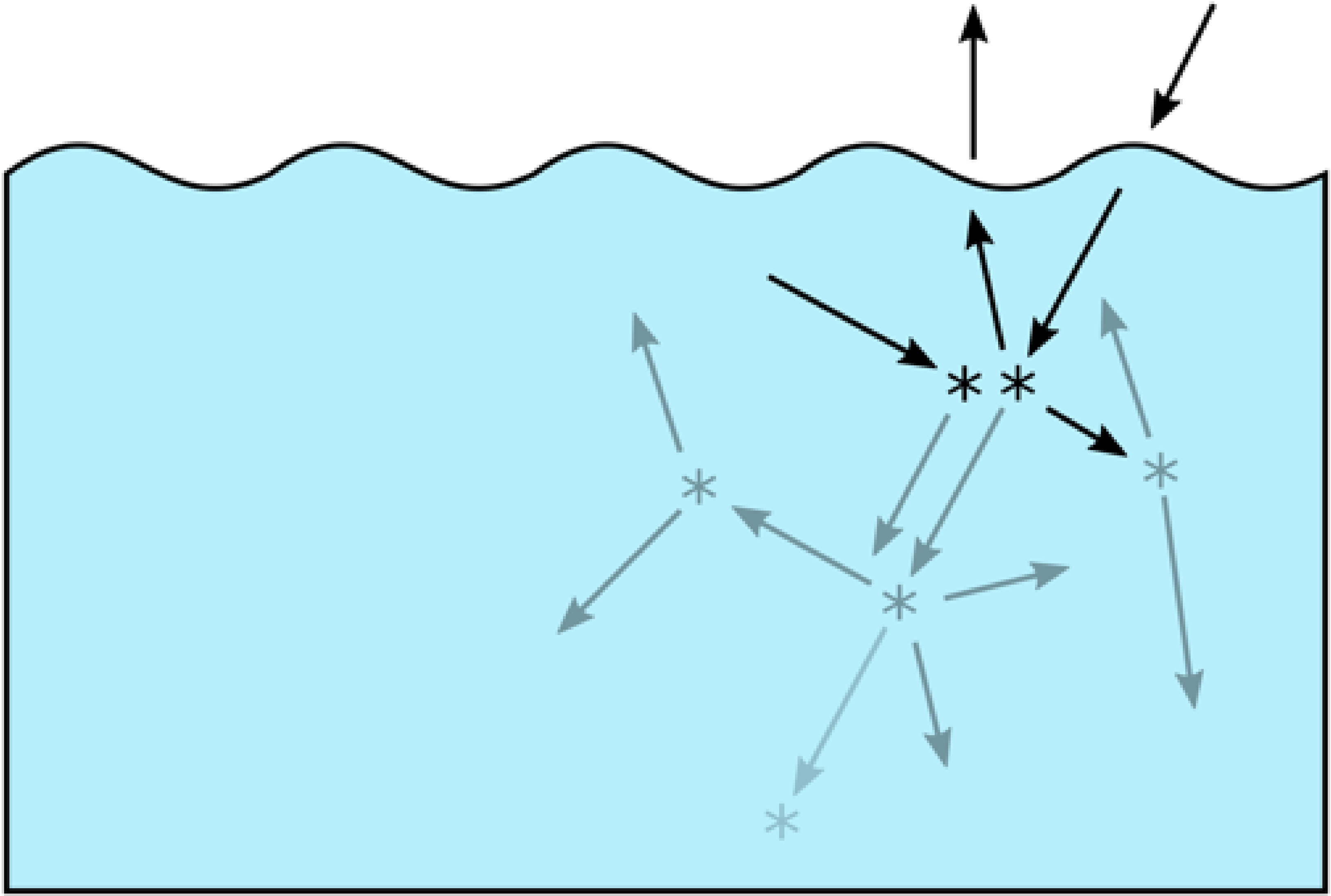
Losses due to absorption + scattering = beam attenuation (a + b = c)

In-scattering



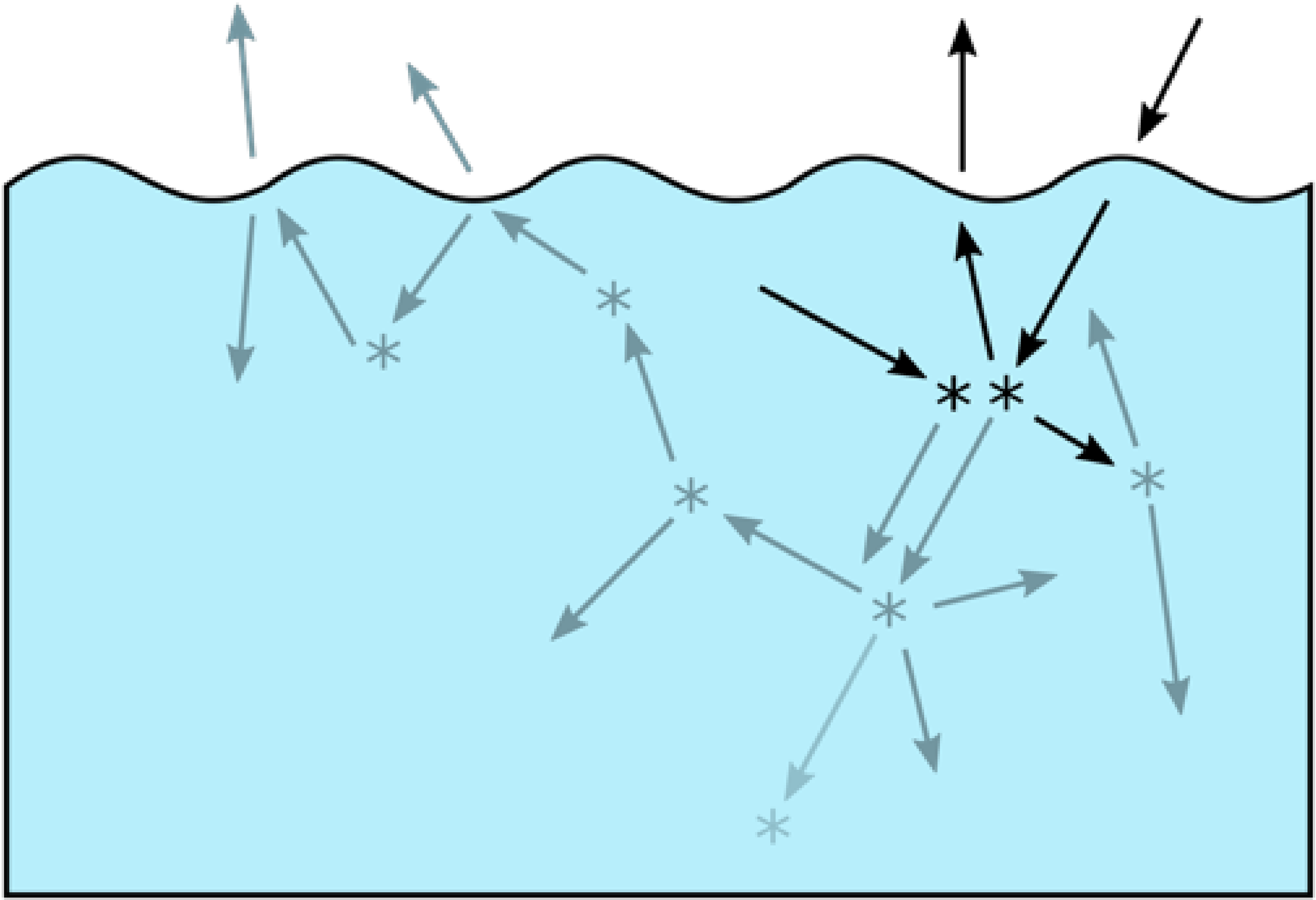
“In-scattering” of light from other directions

Multiple scattering



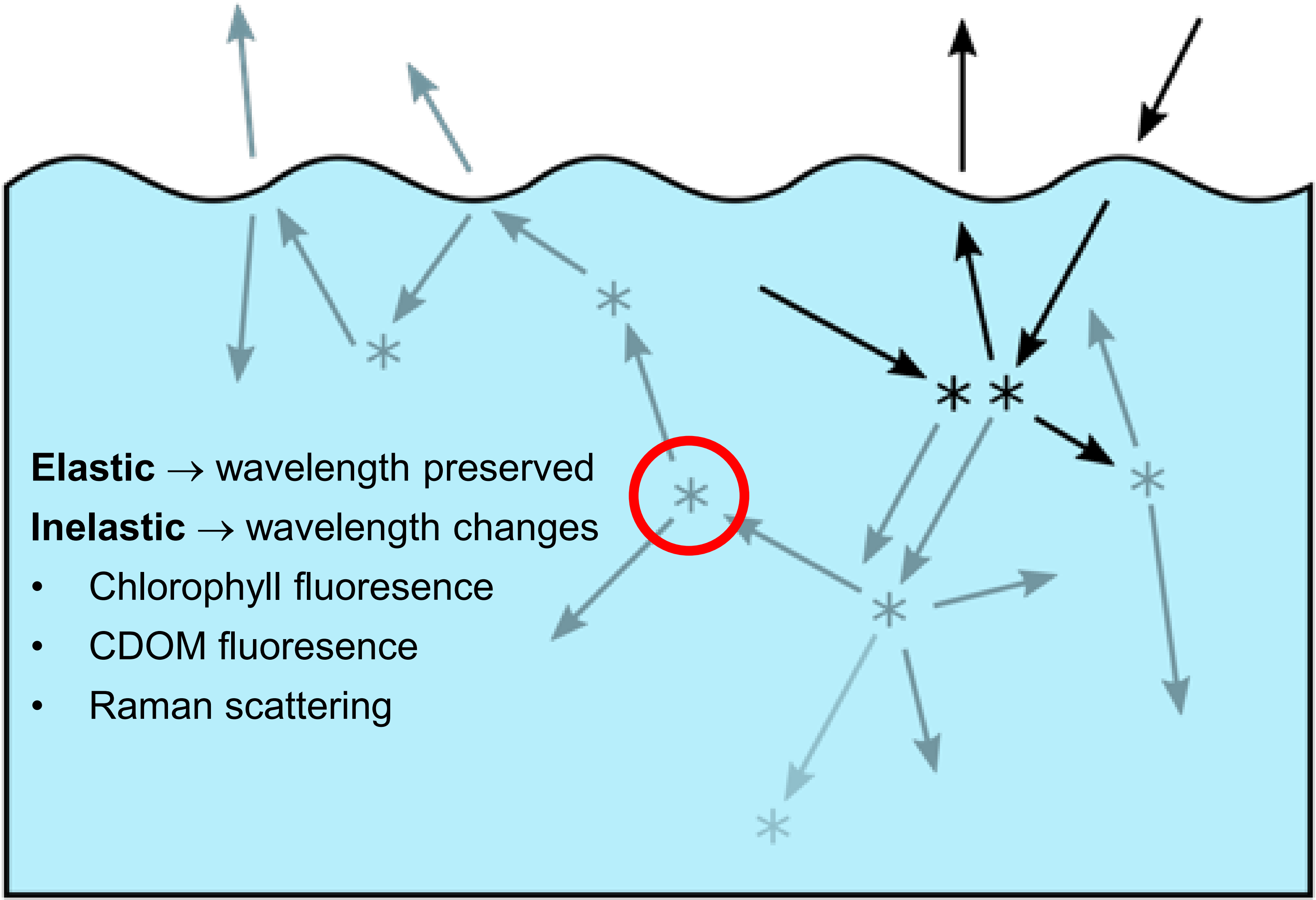
Two orders of scattering in water

Multiple scattering including air-water interface



Multiple orders of scattering including from the water surface underside

Inelastic scattering (fluorescence) vs. elastic scattering



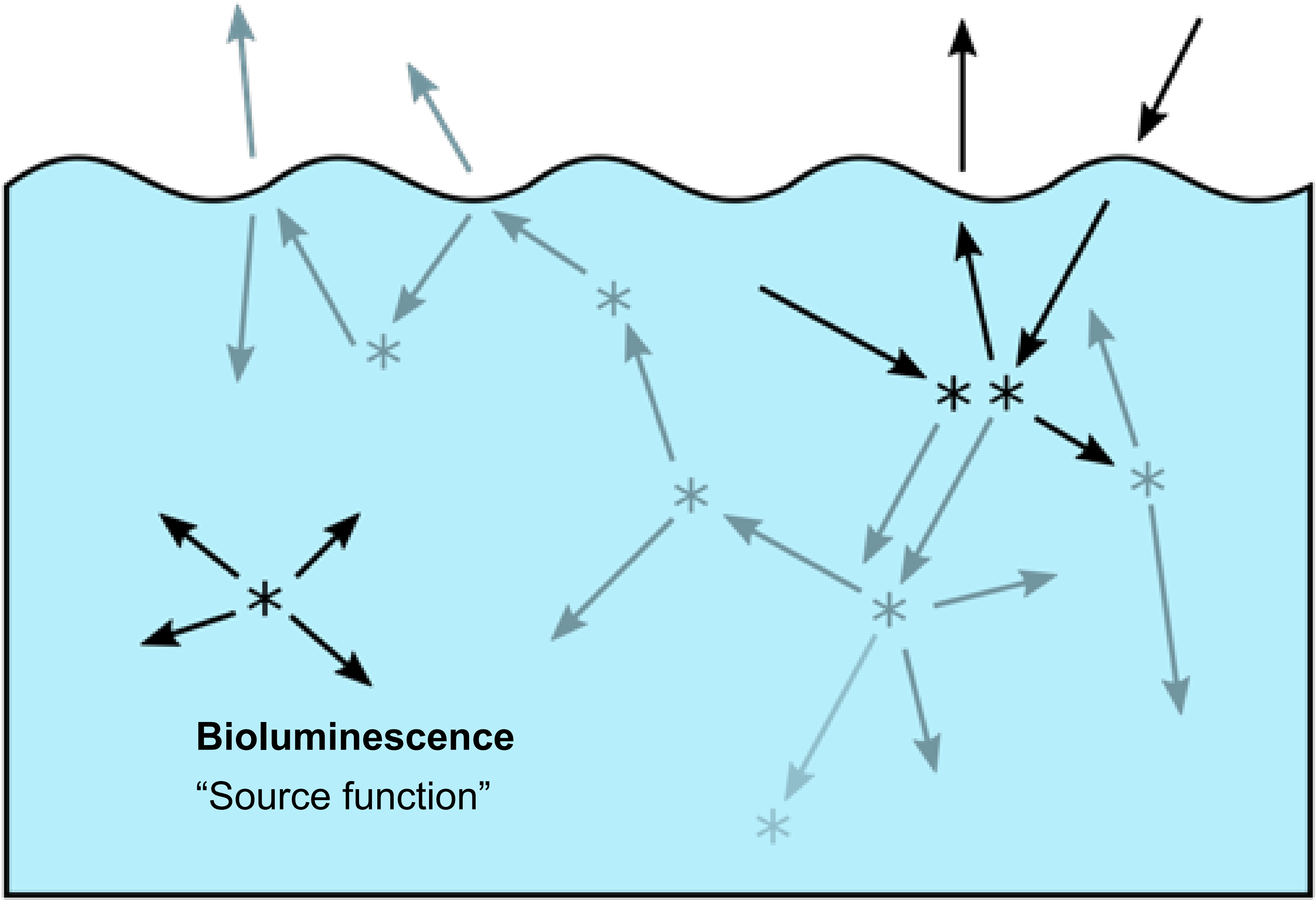
Elastic → wavelength preserved

Inelastic → wavelength changes

- Chlorophyll fluoresence
- CDOM fluoresence
- Raman scattering

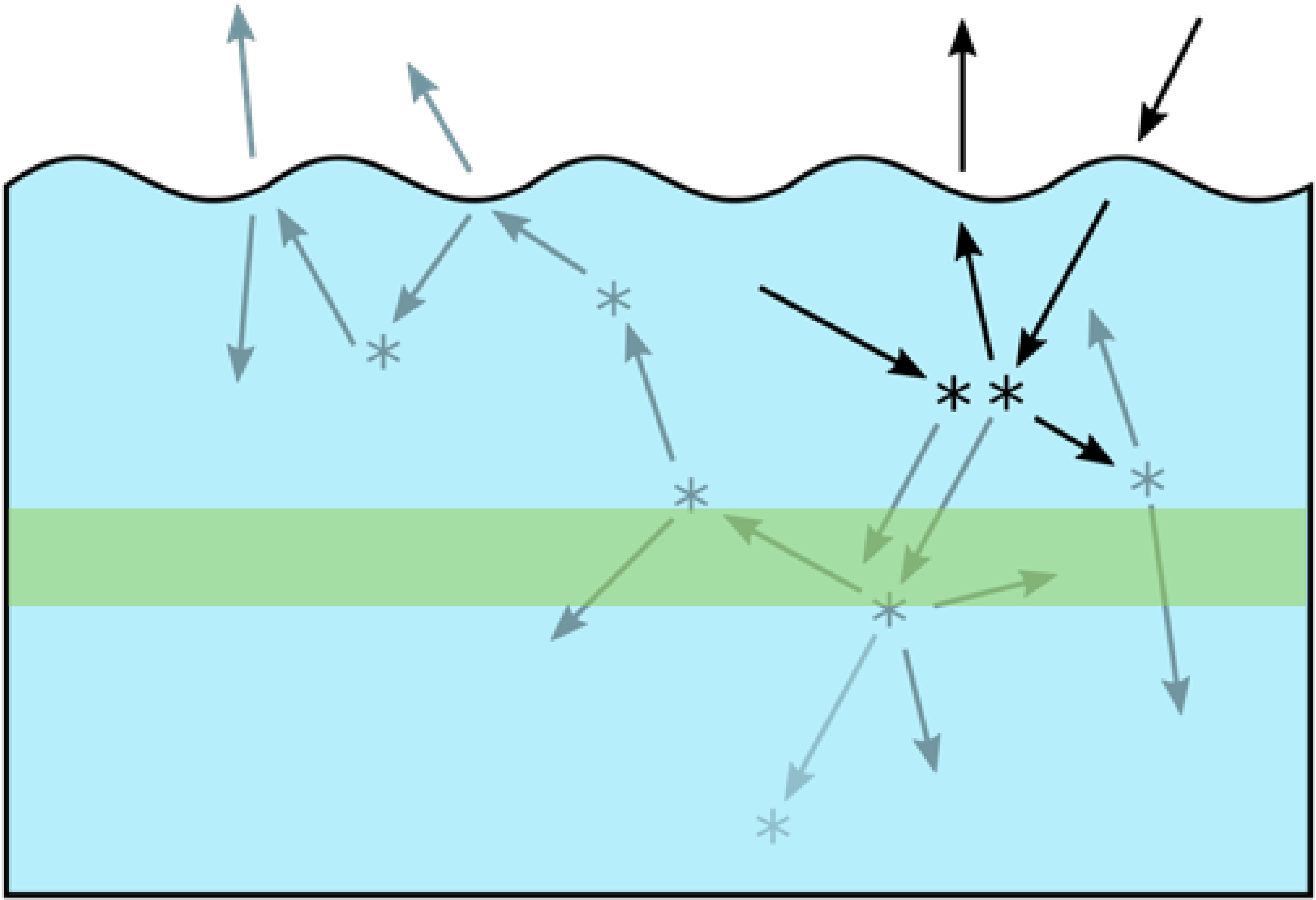
Typically wavelength gets longer (loss of energy) (but not always!)

Bioluminescence



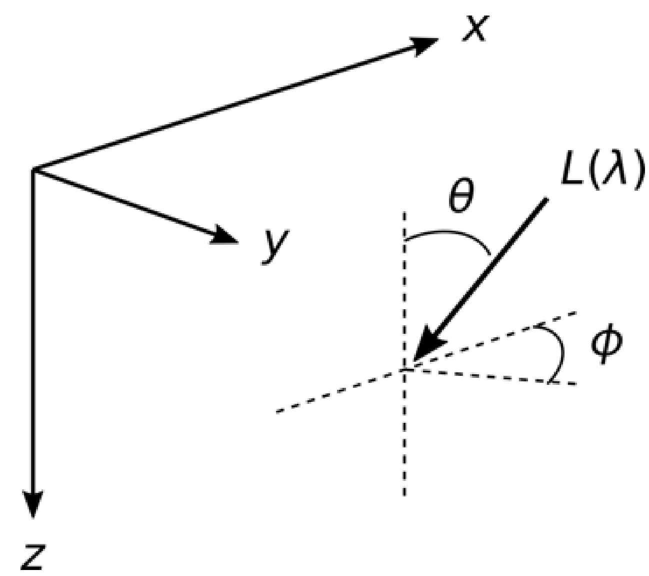
A source of light within the system

Vertical structure



Example – a layer at depth, some paths interact with it some do not

The Complete Solution



Would be:

The radiance distribution (*L*)

- In every direction
- At every point in space
- For each wavelength

I.e.

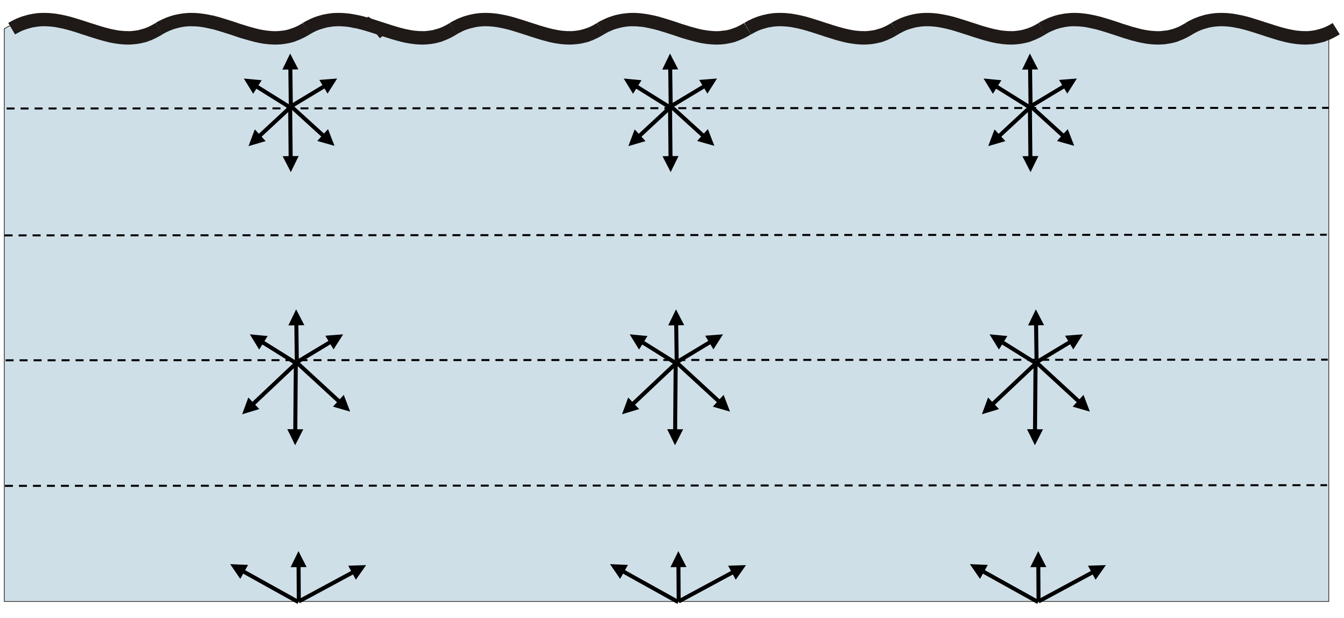
$$L(x, y, z, \theta, \phi, \lambda) \qquad (\mathrm{Wm^{-2}sr^{-1}nm^{-1}})$$

From the complete radiance distribution every other radiometric quantity or property can be derived

→ irradiances, reflectances, diffuse attenuation coefficients (*K* values), etc.

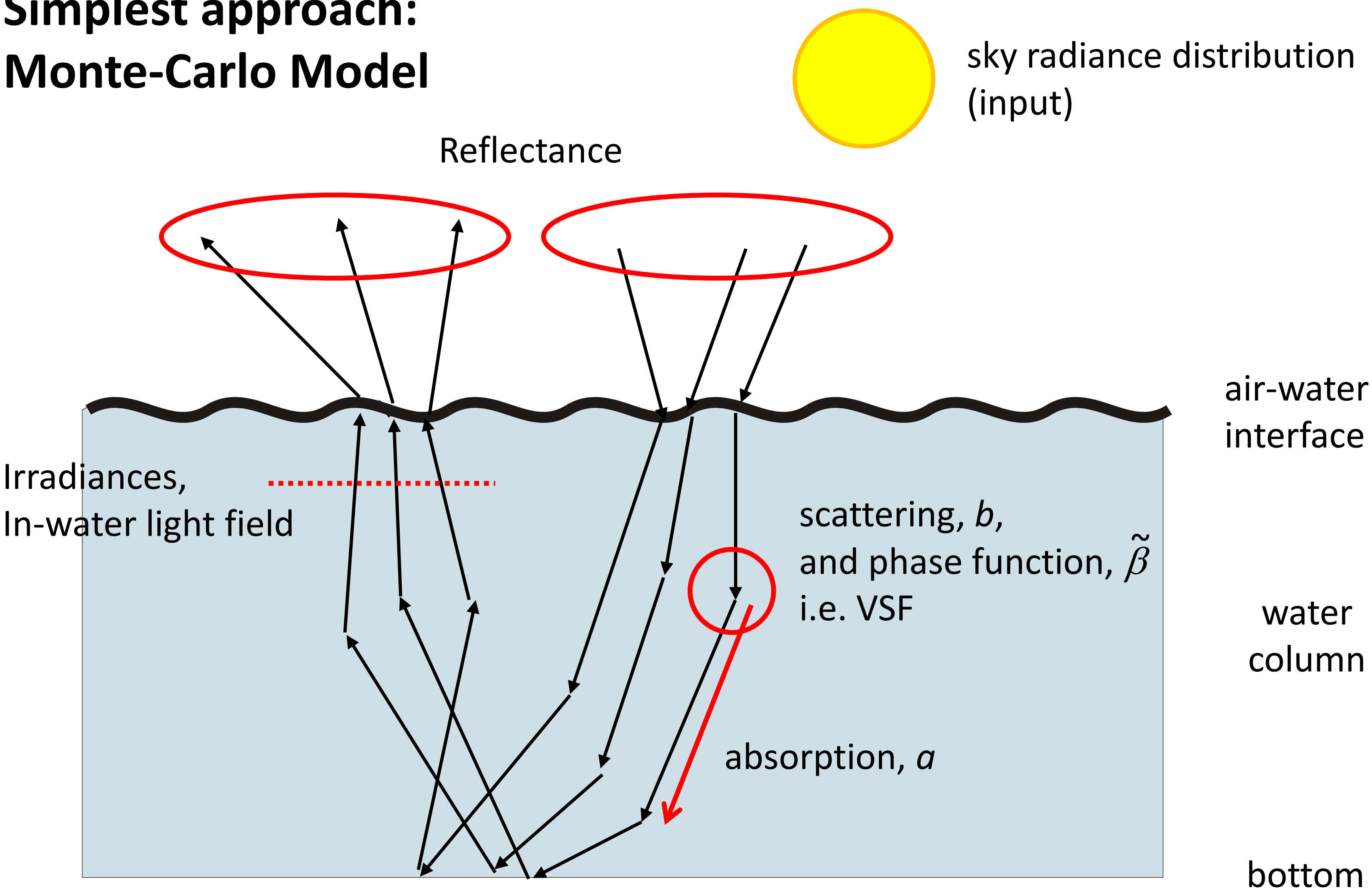
More efficient solution - First step, plane parallel model

- Assume radiance distribution is the same across horizontal planes
- It does not depend on x and y (horizontal position)
- 3D problem becomes 1D
- Reasonable approximation for deep water or homogenous bottoms



so now we want to determine this $L(z, \theta, \phi, \lambda)$ ($Wm^{-2}sr^{-1}nm^{-1}$)

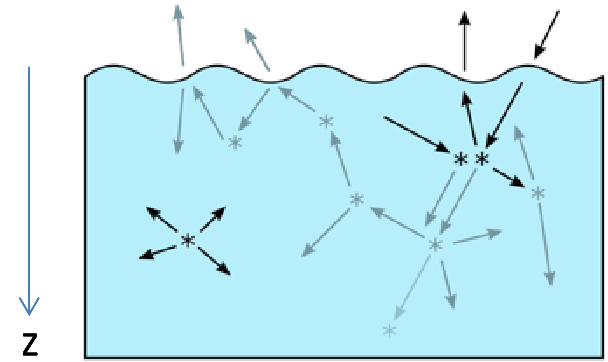
Simplest approach:
Monte-Carlo Model



→ Close association between implementation and physical concepts

→ But, **inefficient** and subject to **statistical noise**

Radiative Transfer Equation (RTE)



Change in radiance due to scattering and absorption when moving in +z can be captured by an equation.

One-dimensional, time independent, scalar RTE,
no inelastic scattering, randomly orientated particles

$$\cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} = -c(z,\lambda)L(z,\theta,\phi,\lambda) + b(z,\lambda)\int_0^{2\pi}\int_0^{\pi} L(z,\theta,\phi,\lambda)\tilde{\beta}(z,\theta',\phi' \rightarrow \theta,\phi,\lambda)\sin\theta' d\theta' d\phi' + S(z,\theta,\phi,\lambda)$$

attenuation

in-scattering

additional sources

What we want to know is: $L(z, \theta, \phi, \lambda)$ (as underlined in red)

This describes how the full directional radiance distribution changes as you take a small step down through the water column (i.e. *z* increases).

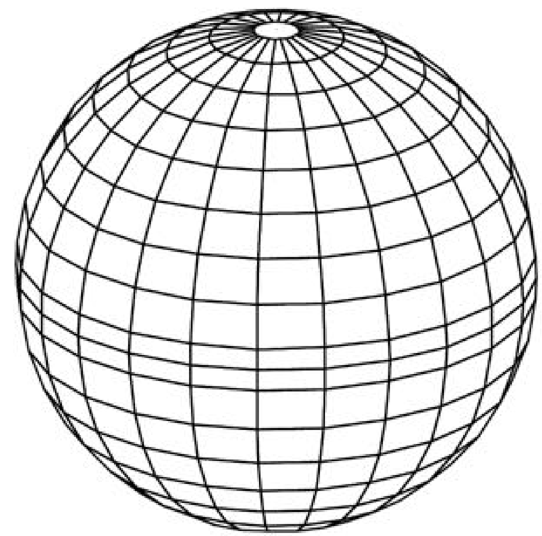
Discretisation of direction

HydroLight standard discretisation

Resolution is $10^\circ \times 15^\circ$

Full sphere of directions 18 x 24 quads plus end-caps = 434 entries

Works with quad averaged radiances



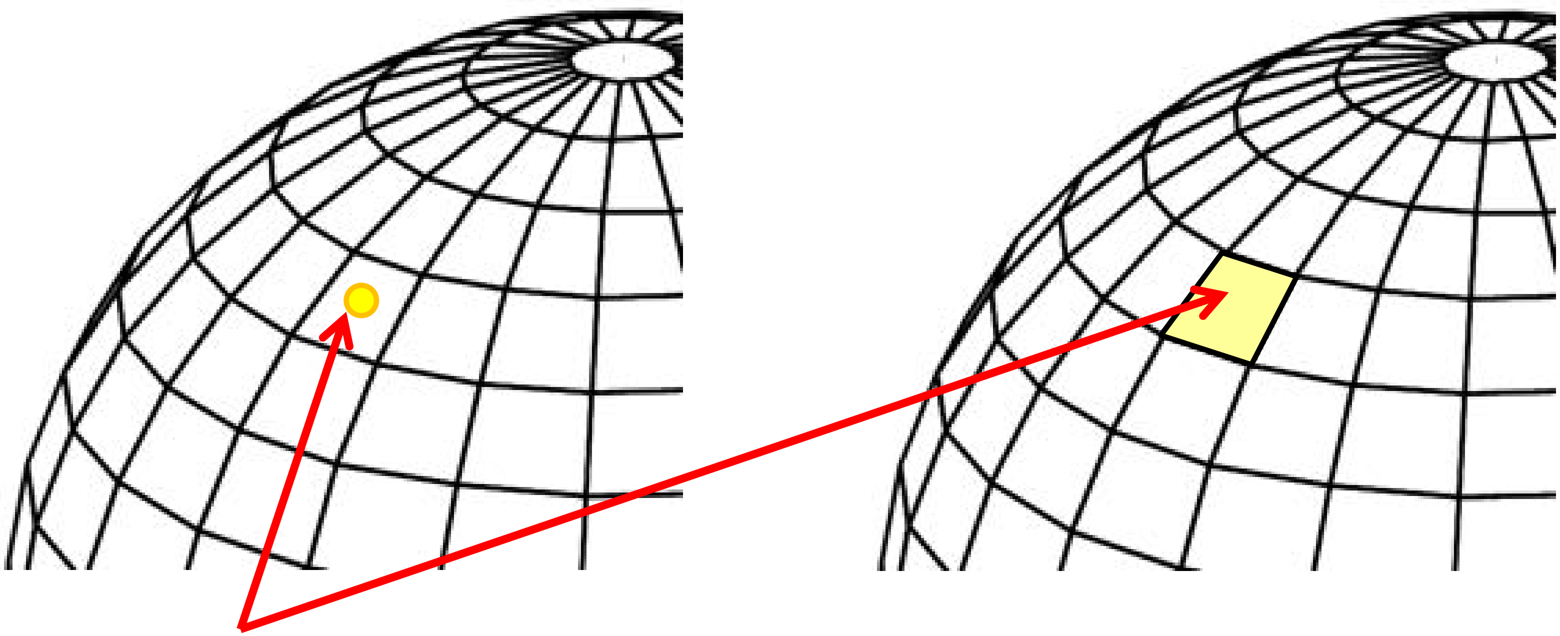
$L(z, \theta, \phi, \lambda) \Rightarrow$ a table of 434 numbers for any particular z and λ

- Other numerical models use different methods but all reduce the radiance over the sphere of all directions to a finite number of values.
- Some kind of 'angular resolution' but not always clear what it is.
- Advantage of quad-averaged $\rightarrow$ easy to compare to Monte-carlo model.

Quad-averaged radiances (HydroLight)

The solar disc is smaller than one quad

So one consequence is that the direct solar radiance is spread over the quad

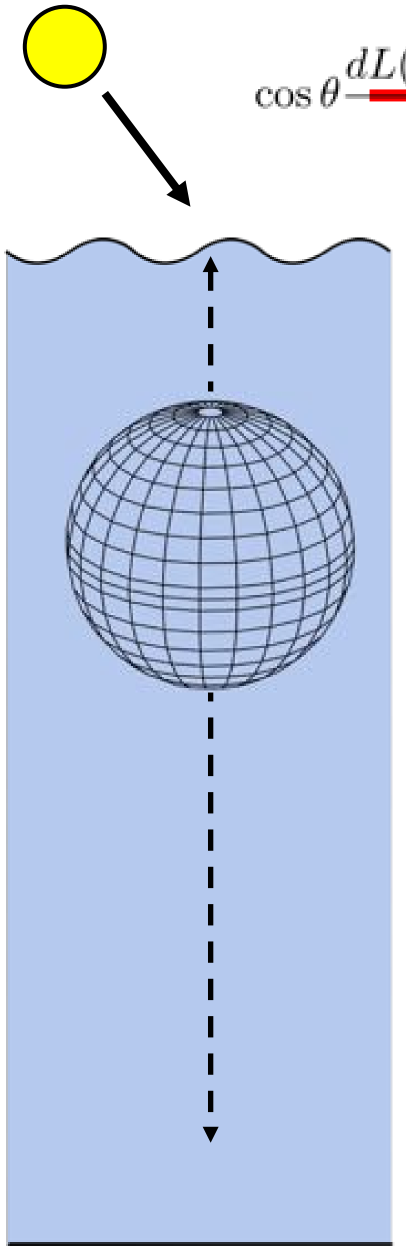


However the total energy as averaged over the quad is the same in both cases and correct.

Makes almost no difference to most quantities of interest, due to scattering the direct radiance is rapidly spread out underwater anyway.

See Tech Note: HTN2_AngularResolution.pdf

Why the RTE is non-trivial to solve



$$\cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} = -\,c(z,\lambda)L(z,\theta,\phi,\lambda) + b(z,\lambda)\int_0^{2\pi}\int_0^{\pi} L(z,\theta,\phi,\lambda)\tilde{\beta}(z,\theta',\phi' \rightarrow \theta,\phi,\lambda)\sin\theta' d\theta' d\phi' + S(z,\theta,\phi,\lambda)$$

Why can't we just start with the table of radiances at the top of the water column and propagate it down?

Problem – at the top of the water column we only know the radiances downwards.

We need to get to the bottom boundary before we can fully establish the relationship between the upward and downward radiances.

Bottom boundary reflectance, R_b or R_∞

Methods to solve the RTE

- Monte-Carlo models — stochastic – noise
- Successive orders of scattering (e.g. OSOAA) — 'convergence' approximation
- Discrete ordinates method (e.g. DISORT)
- Invariant Imbedded method (HydroLight)

'RTE solvers'

Differences:

How they represent the radiance over the sphere of all directions.

How they solve the RTE.

Homogenous layers or continuous profiles (fundamental design).

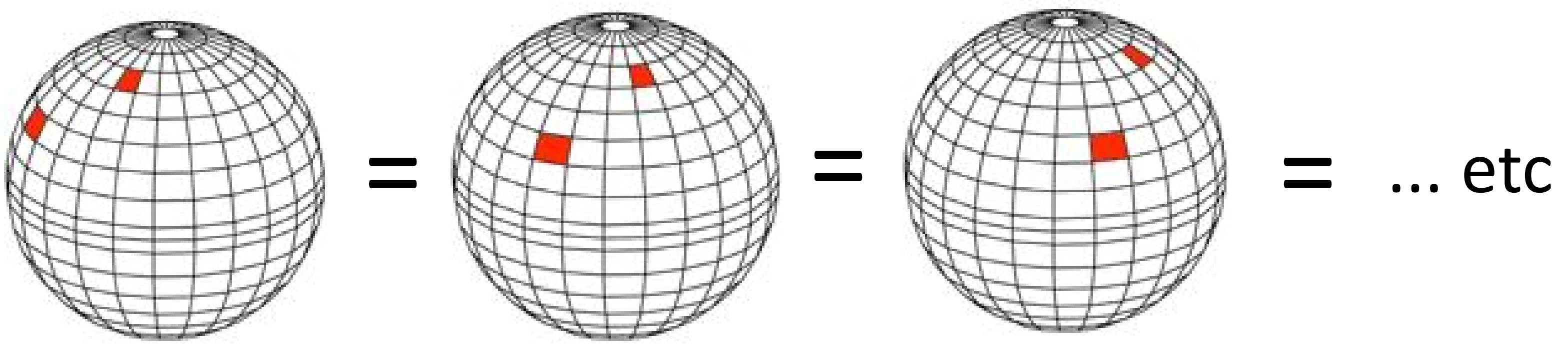
Some implementations are vectorial some not (but all could be).

* Many other implementations exist for atmospheric and planetary optics

Exploiting symmetries

2. Exploit the fact that the **phase function is dependent on relative angle only**.

I.e. the relationships between these pairs of quads are equivalent:



It means the matrices used in the calculation can be transformed into matrices with lots of zeros in them – more computationally efficient.

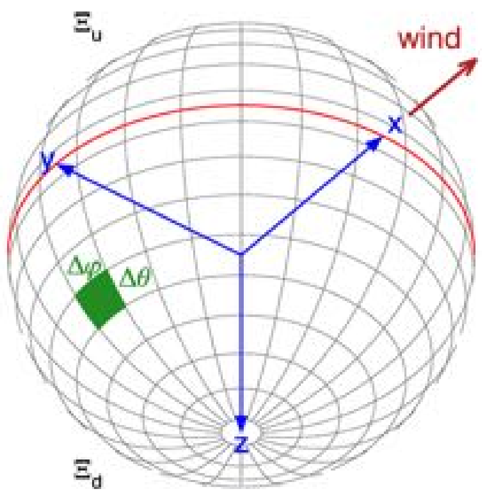
See the Ocean Optics Webbook or Mobley's *Light and Water* (1994) for details.

All the numerical methods exploit this.

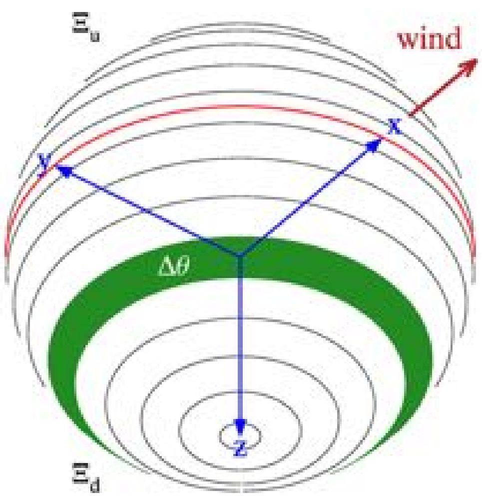
Fully azimuthally-averaged solution – EcoLight

EcoLight uses **azimuthally averaged radiances** within solid angle bands.

HL



EL



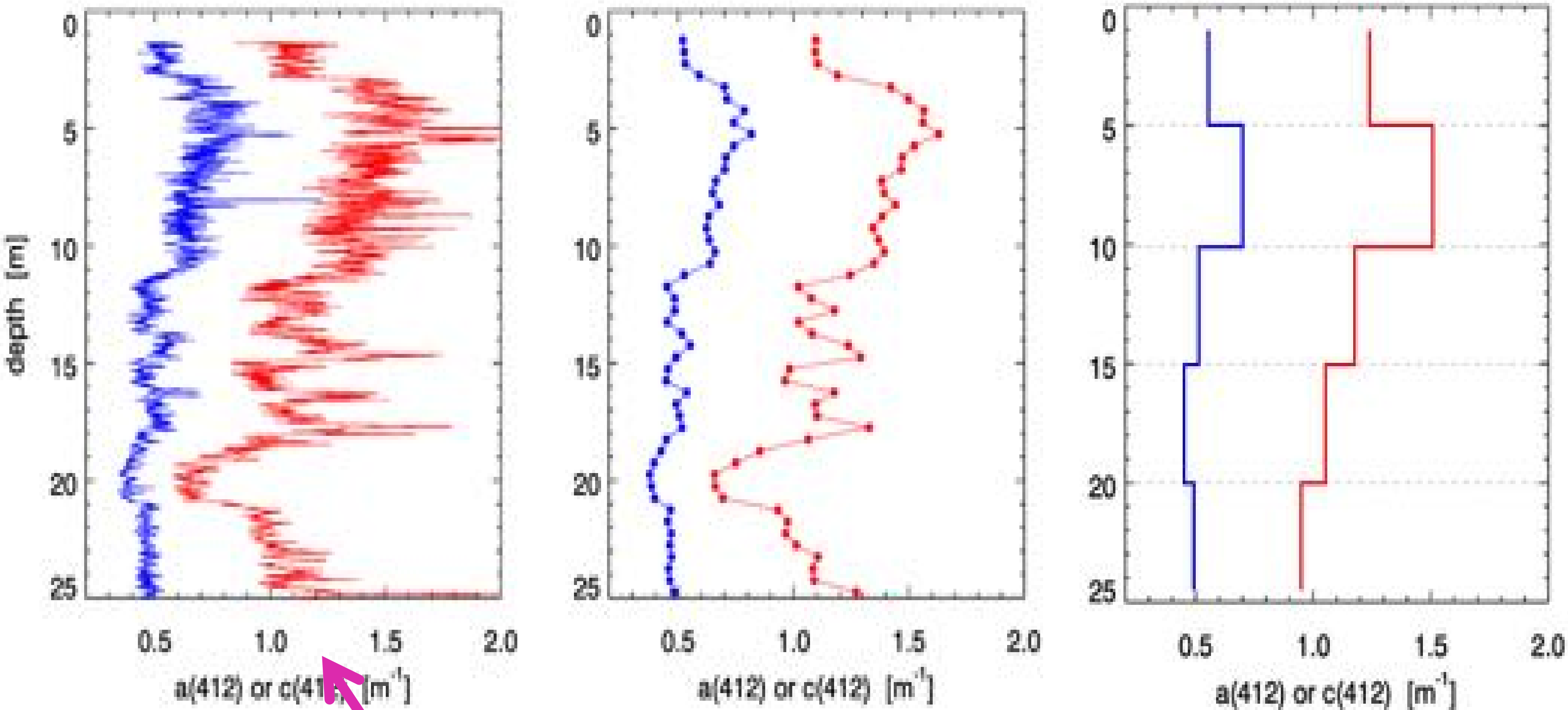
Irradiances and **polar cap radiances** come out the same for HydroLight and EcoLight.

Diffuse attenuations (*K*-values), **reflectances** R, R_{rs}, etc. are also the same.

- EcoLight is typically 20 to 1000 times faster than HydroLight.

Profile Data

Measured AC-S data | Binned data for HydroLight | Discrete layer based models



Don’t give this to HydroLight!

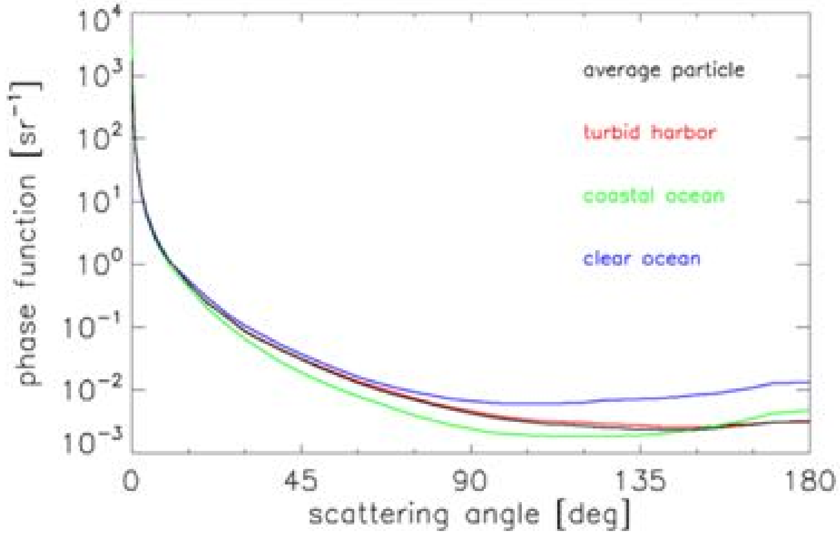
absorption coefficient, *a*

attenuation coefficient, *c*

Phase functions in HydroLight

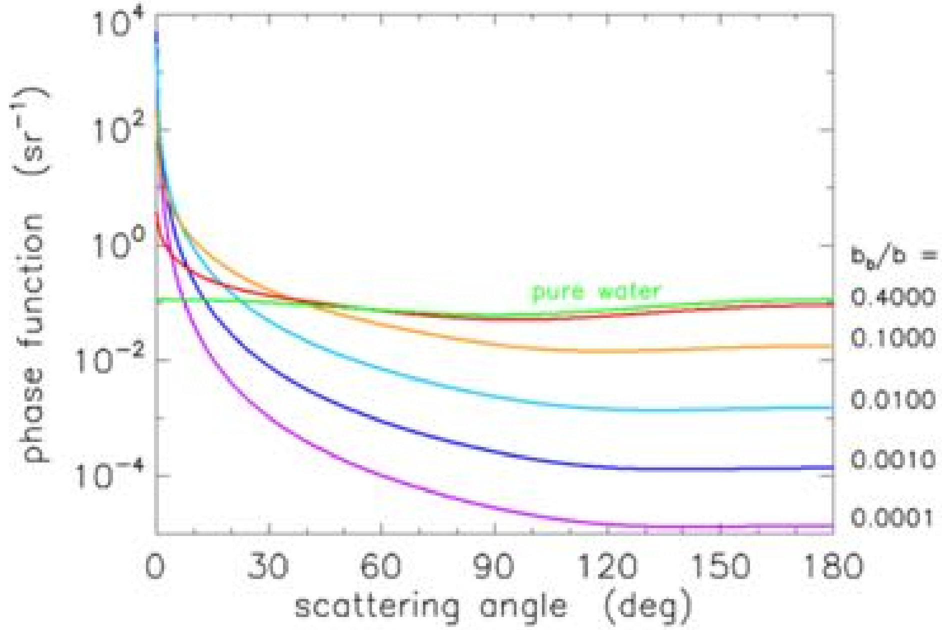
1) **Petzold's** phase function

- Empirical
- Assumed the same everywhere
- Backscatter ratio $b_b/b = 0.0183$



2) **Fournier-Forand** phase function

- Theoretical
- Shape depends on backscatter ratio b_b/b

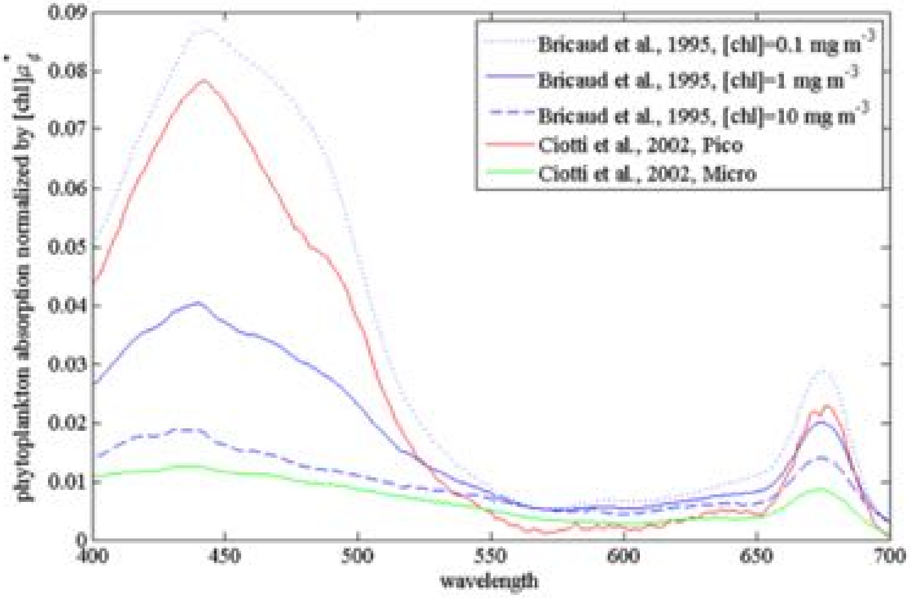


3) **Measured or modelled**

- Provide your own!

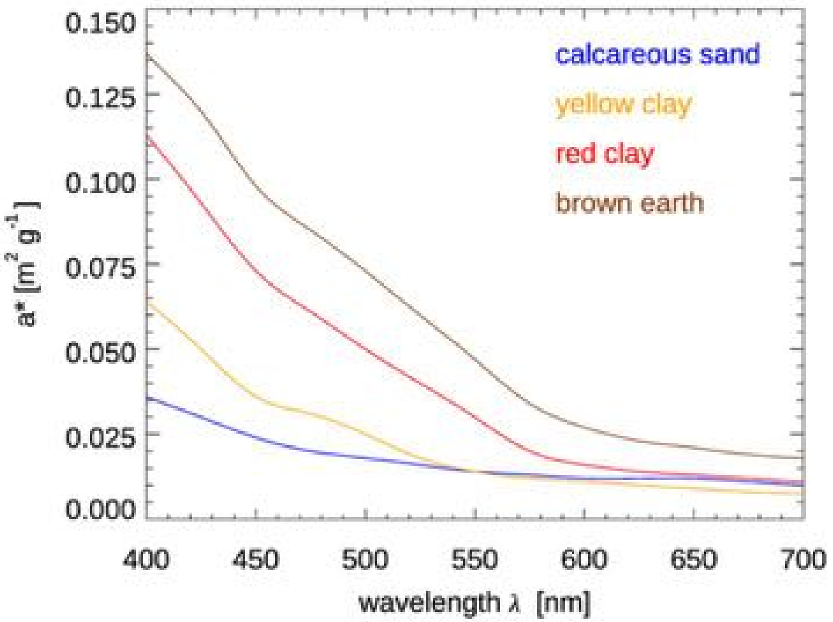
IOPs from bio-optical models - *examples!*

Chl-specific phytoplankton absorption



[From ocean optics web-book]

Concentration-specific mineral absorption



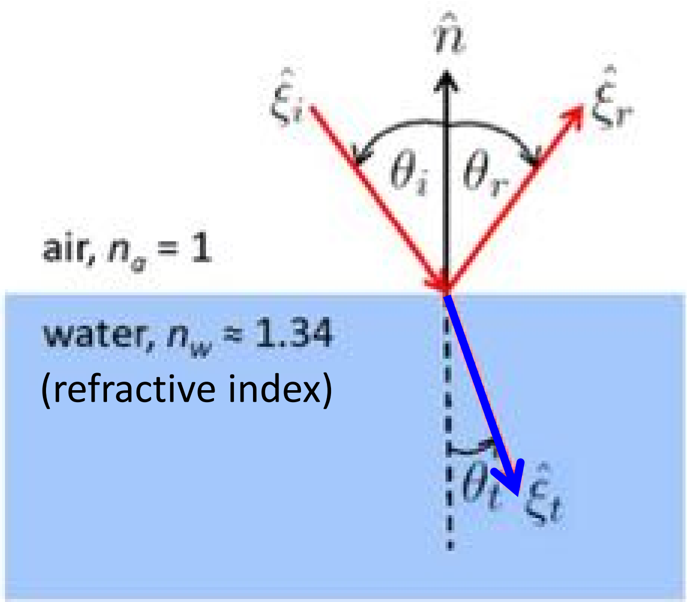
[Ocean optics web-book, Ahn, 1999]

- HydroLight and other models have built-in bio-optical models *for convenience*
- These are just examples!
- Your specific location may deviate from these models, always be critical!

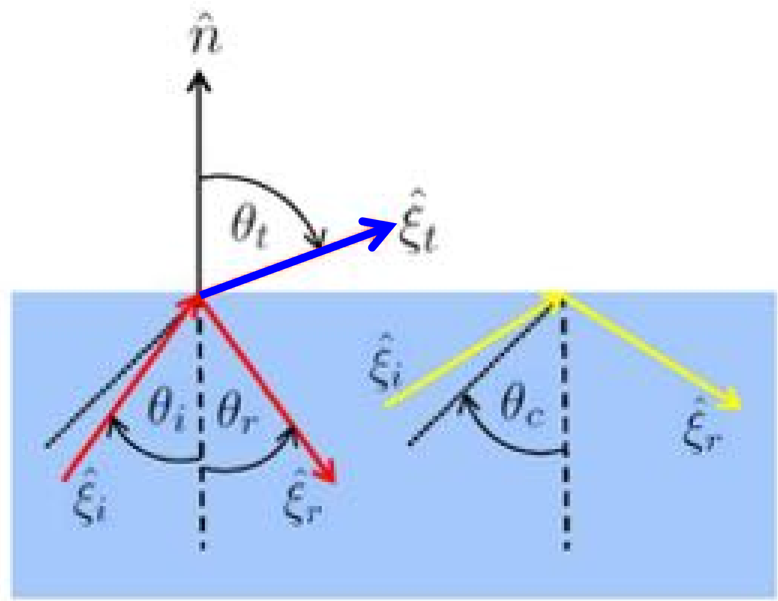
Garbage in = garbage out!

Air-water interface – flat surface (Fresnel equations)

air incident

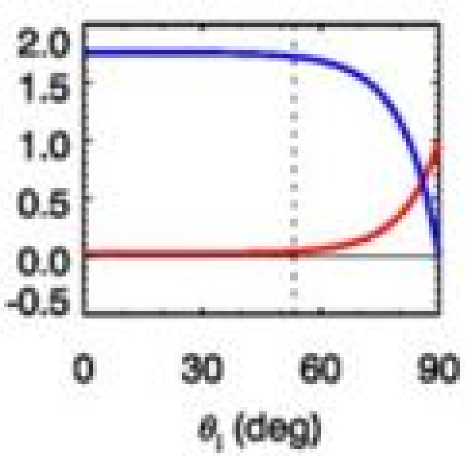


water incident



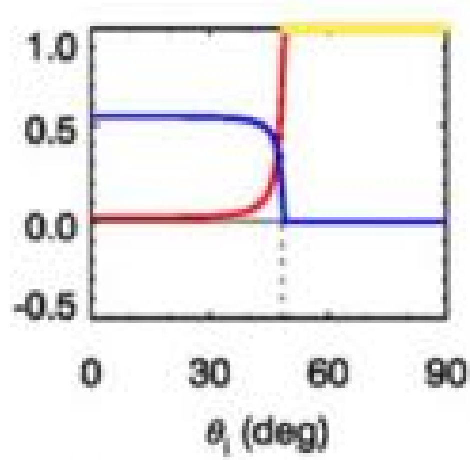
transmission

reflectance



transmission

reflectance



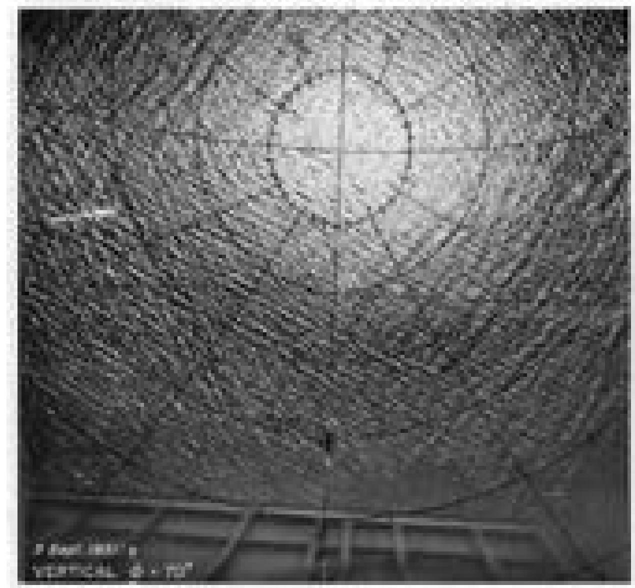
total internal reflectance

Cox-Munk wind-speed wave-slope law

Cox and Munk equations

- 1950s - based on photographs of surface glitter
- Many subsequent studies: all agree

Cox & Munk (1956) Slopes of the Sea Surface Deduced from Photographs of Sun Glitter. *Scripps Inst. Oceanogr. Bull*. 6(9): 401–88



Result is statistical model of the slopes of the sea surface:

U_{10} = wind speed ms^{-1}

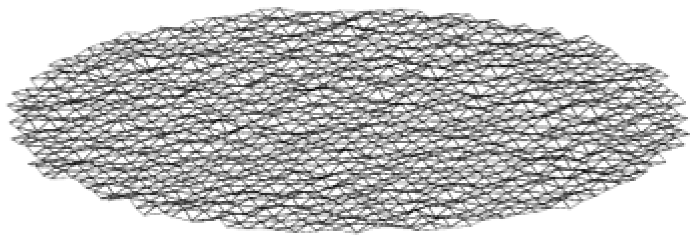
Mean square slope (along wind) $\sigma_w^2 = 0.00316\, U_{10}$

Mean square slope (cross wind) $\sigma_c^2 = 0.00192\, U_{10}$

- Predicts bigger slopes in the wind direction (wave peaks and troughs).
- Works only at large scales and open seas (mature seas).
- Says nothing about wave heights or the coherent structure of the surface.

Air water interface modelling

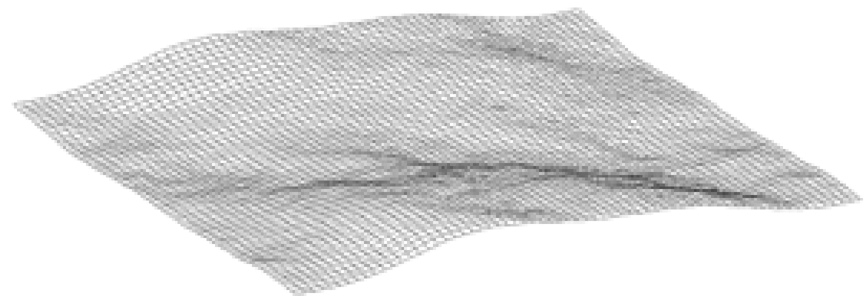
Cox-Munk slope statistics only



scale invariant

No wave structures

Slope and elevation statistics



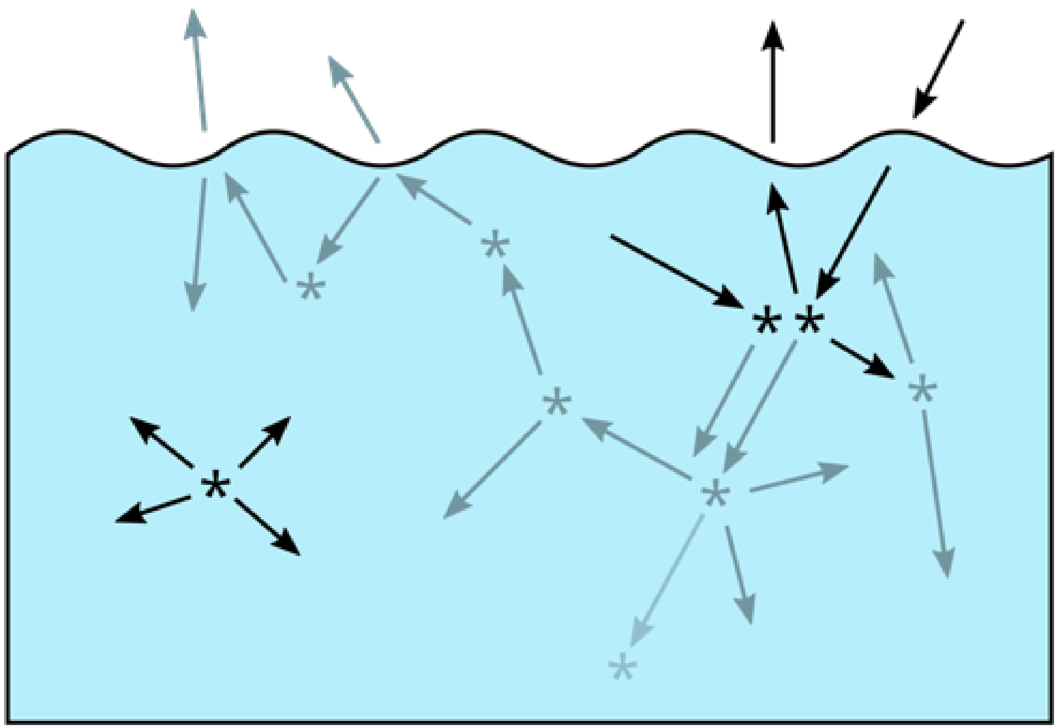
3 m

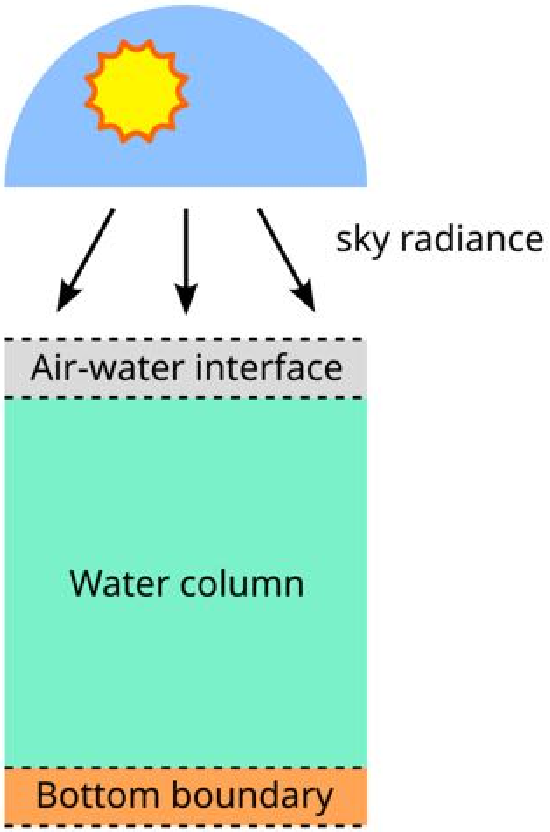
Needs to be large and detailed enough to cover features from 100s m to millimetres

- Ray tracing is used to characterise the directional reflectance and transmittance - pre-calculated functions.
- See Kay et al. (2011) *Optics Express*, 19, 6493.

'Uncoupled' or separate ocean-atmosphere model

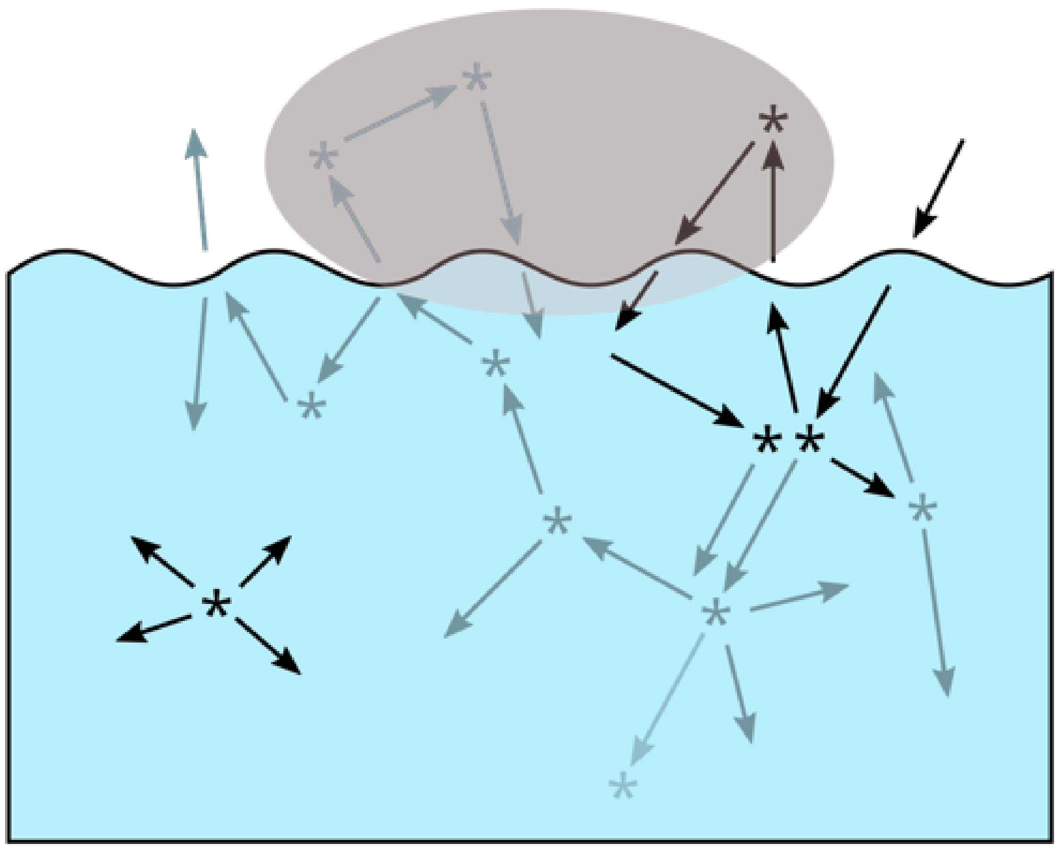
- Sky radiance above the water is from an independent model or data
- RADTRAN-X model is part of HydroLight

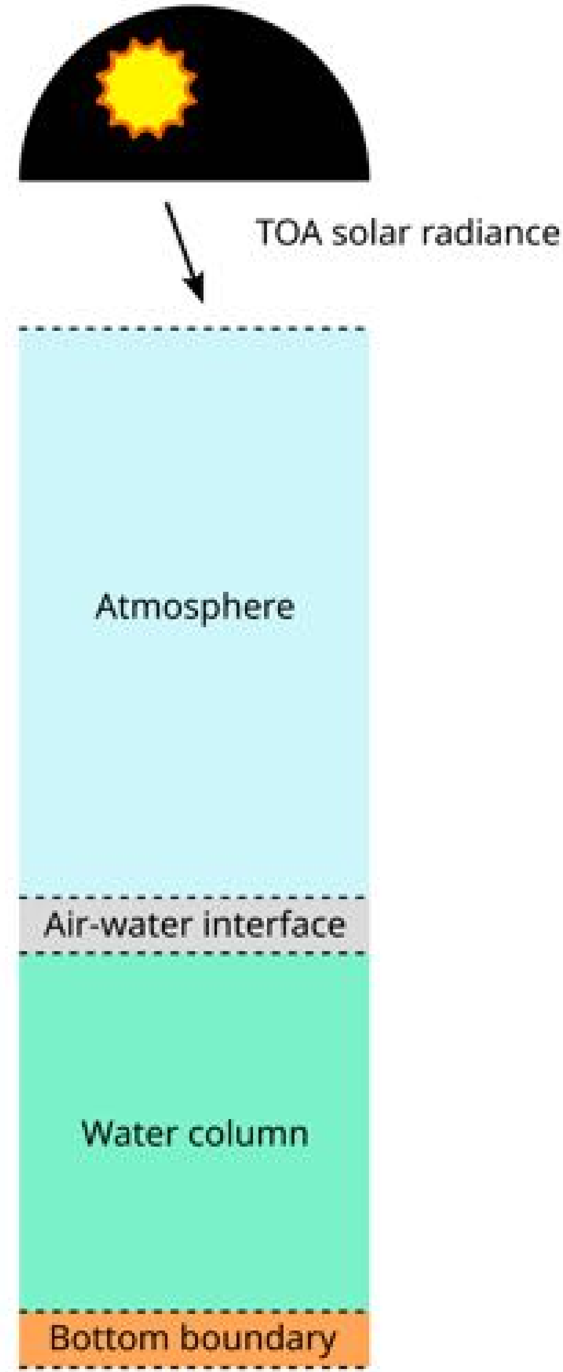


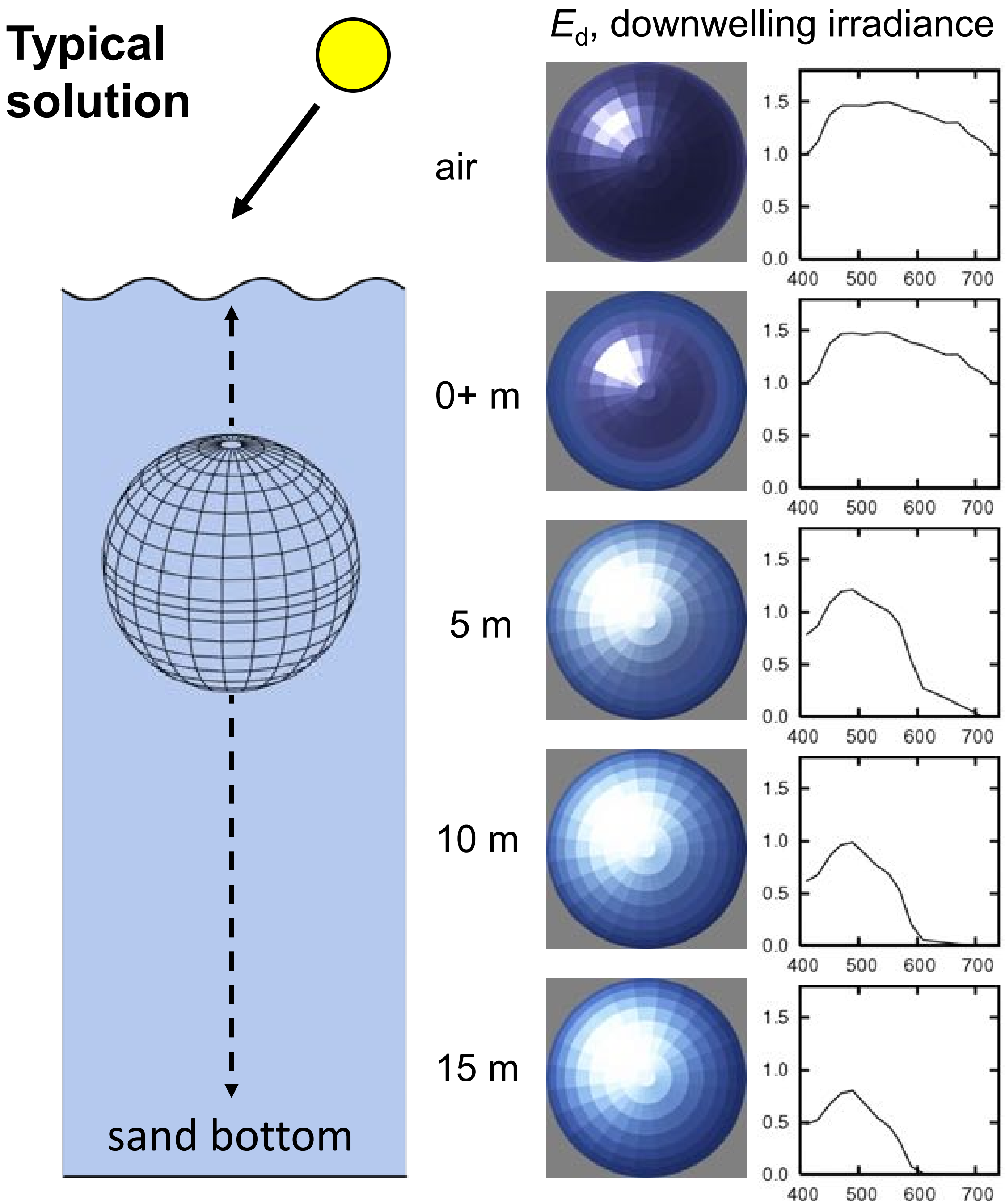


Coupled ocean-atmosphere model

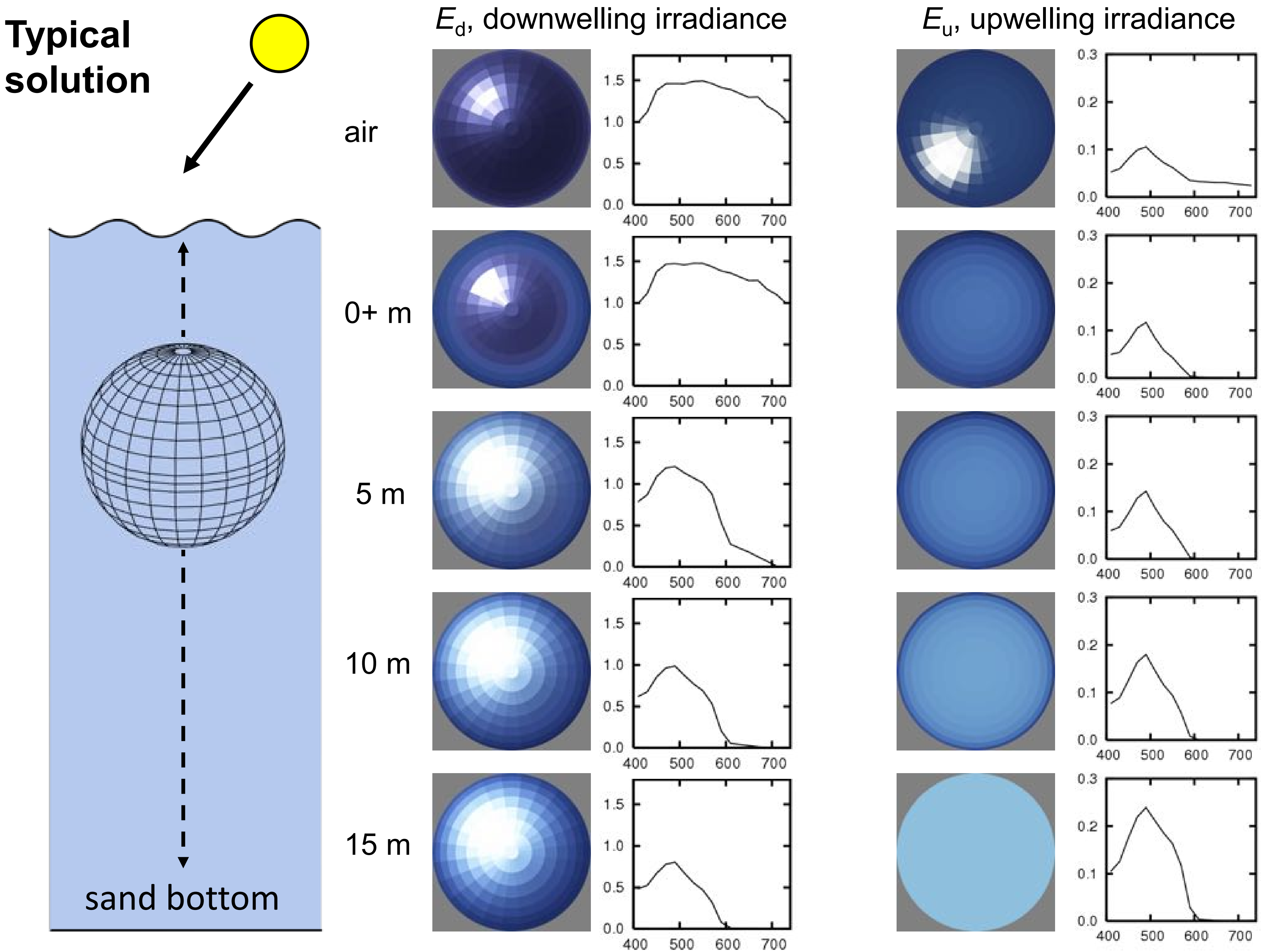
- Single model for atmosphere and water column
- Includes paths that leave the water surface and scatter back down





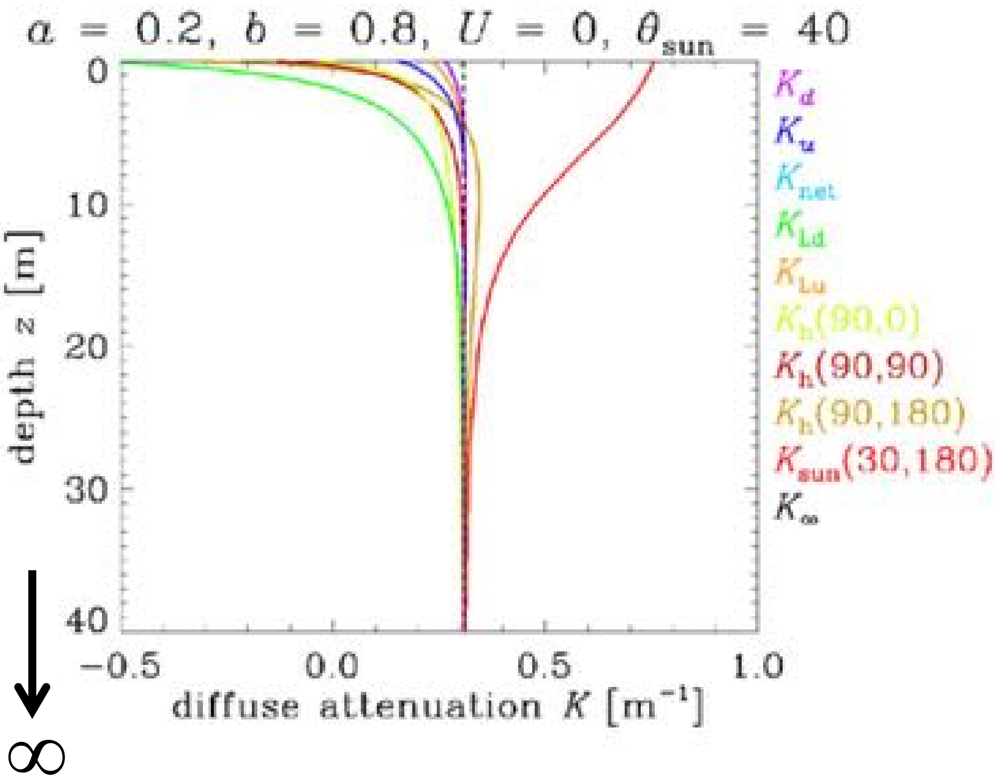


Typical solution
E_d, downwelling irradiance
air
0+ m
5 m
10 m
15 m
sand bottom
1.5
1.0
0.5
0.0
400
500
600
700



Typical solution
E_d, downwelling irradiance
E_u, upwelling irradiance
air
0+ m
5 m
10 m
15 m
sand bottom

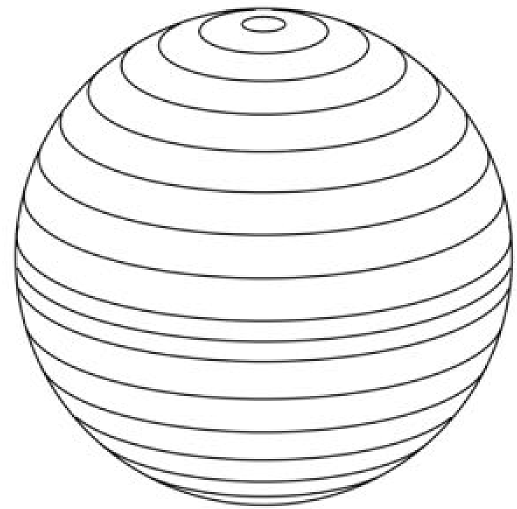
Infinite depth (homogenous IOPs)



Asymptotic radiance distribution

Tends to a constant relative directional distribution of light

→ **azimuthally constant**



<u>Amount</u> of light in each wavelength decreases with depth according to an exponential function.

All *K*-values at a given wavelength become the same.

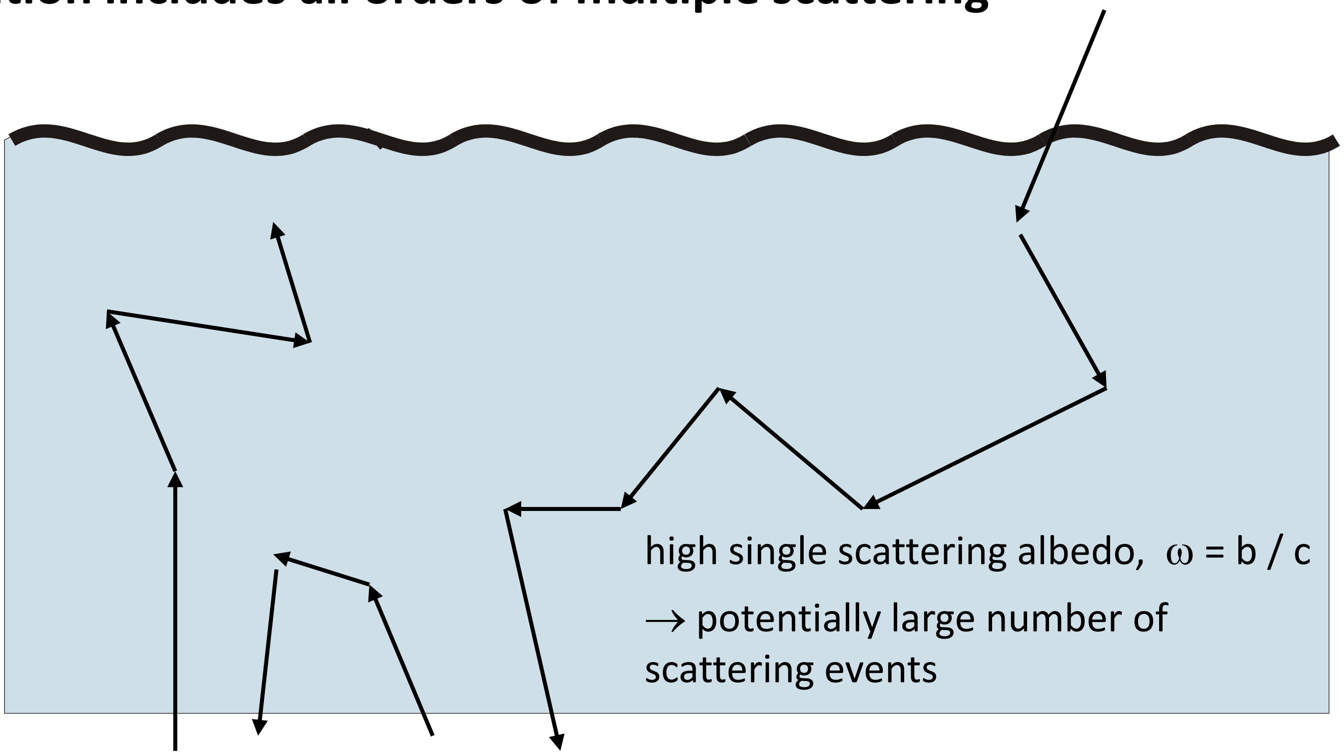
Asymptotic *K*-values are IOPs!

Comparison of HydroLight vs. Monte Carlo

- **Run time linearly proportional to optical depth** (attenuation × physical depth)
  Monte Carlo ∝ exp(optical depth)

- **Run time independent of IOP(z) complexity, arbitrary depth resolution**
  not a set of homogeneous layers as used in some methods

- **Solution includes all orders of multiple scattering**



HydroLight current summary of features and limitations

- Time independent / time averaged
- One spatial dimension (depth) - no restrictions on depth dependence of IOPs (not a "layered" model)
- No restriction on wavelengths, included data from 300 to 1000 nm
- Model for sky radiance onto sea surface, or can load arbitrary data
- Air-water interface model (parameterizes gravity & capillary waves via the wind speed)
- Infinite depth or supplied bottom reflectance are possible options
- Includes all orders of multiple scattering
- Includes Raman scatter by water
- Includes fluorescence by chlorophyll and CDOM
- Includes internal sources (bioluminescent layers)
- Polarization not included (may give errors in computed radiances of up to ~10%, ~1% in irradiances)
- Can run from GUI or from scripts

"Validation" - general discussion

What does it mean?

Probably,

"Comparison of model outputs to empirical data are of acceptable accuracy"

Optical Closure:

E.g. measure IOPs → model reflectance → compare to satellite data

<u>**Many different aspects that can be "wrong":**</u>

Physical concepts – plane parallel assumption, scalar approximation

Solution method – e.g. Monte Carlo vs. directionally discretised

Implementation – is the program written correctly, any bugs?

Measurement of empirical data – uncertainties in empirical data

Where is HydroLight on these aspects?

Physical concepts

– physical concepts well accepted within the scope of the model definition
– e.g. scope includes plane parallel assumption, scalar approximation

Solution method

– method is an "exact" physical solution, <u>for quad-averaged radiances</u>!
– numerical issues, only in extreme parameterisations

Implementation

– no serious bugs found in quite a while
– benefit of a long time code-base in use by many people
– bugs are still an ever present danger!

Measurement of empirical data

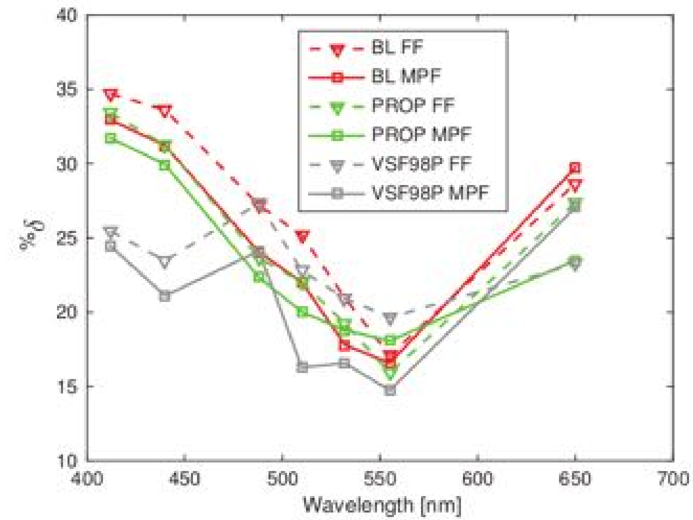
– main area for doubt, both in terms of inputs and output comparisons
– HydroLight includes built-in options, such as phase functions, Chl and CDOM fluorescence, etc. these are empirically based: **USER BEWARE**
– for some real data is scarce, e.g. CDOM fluorescence, only 1 paper!

Examples of optical closure using HydroLight

Tonizzo et al. (2017)
Applied Optics 56, 130-146.

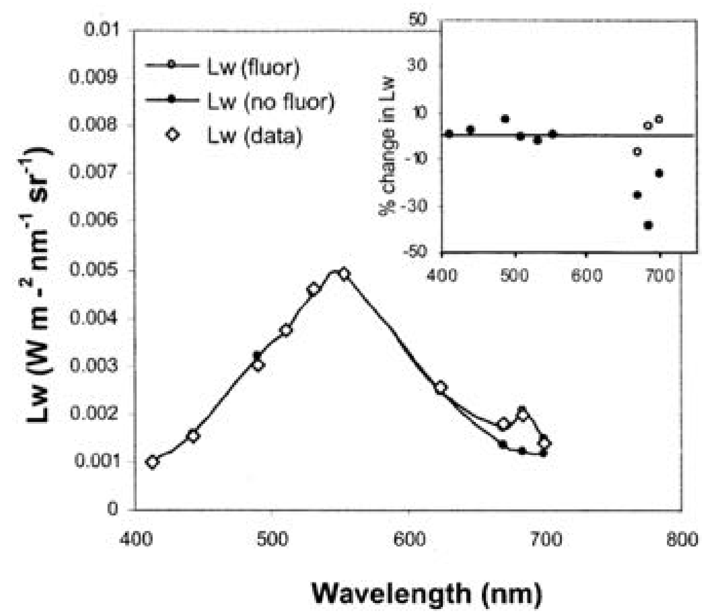
Overal discrepancies between measured R_{rs} and modelled:

- Using measured phase functions ~20%
- Fournier-Forand phase functons ~23%



Tzortziou et al. (2005)
Estuarine, Coastal and Shelf Science 68, 348-362.

Average % difference between modelled and measured water leaving radiances ~7% (0 - 20%)



→ Very careful studies – discrepancies of 20% are in general a very good result.