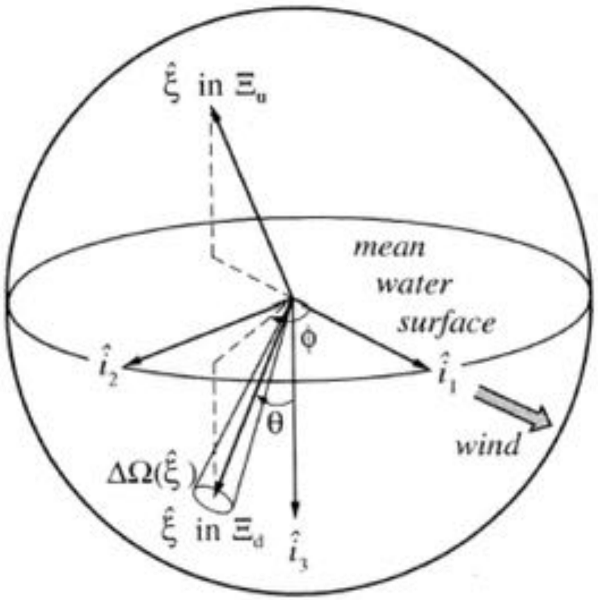
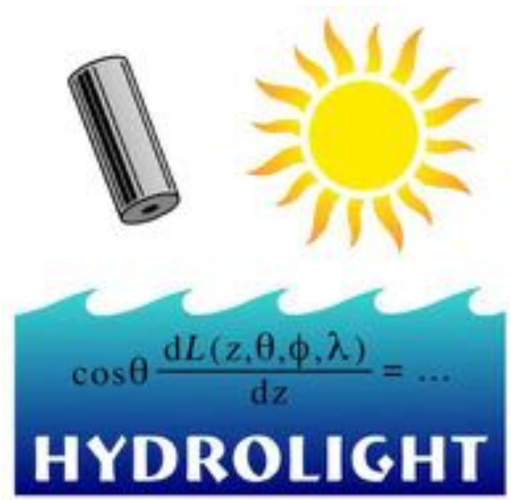
Introduction to Radiative Transfer Theory and Numerical Modelling





John Hedley, IOCCG Summer Lecture Series, 2024

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John Hedley

- Undergraduate Degree – Zoology
- Ph.D. – Remote Sensing of Coral Reefs
- ~10 years in a Coral Reef Ecology Group (Exeter University)
- Since 2012 work through my own company, working with academic and commercial sector.

Professional activities

- Numerical modelling of radiative transfer
- Shallow water remote sensing - coral reefs, seagrasses, satellite derived bathymetry
- Software products, maintain and develop HydroLight
- Benthic photobiology

Numerical Optics Ltd

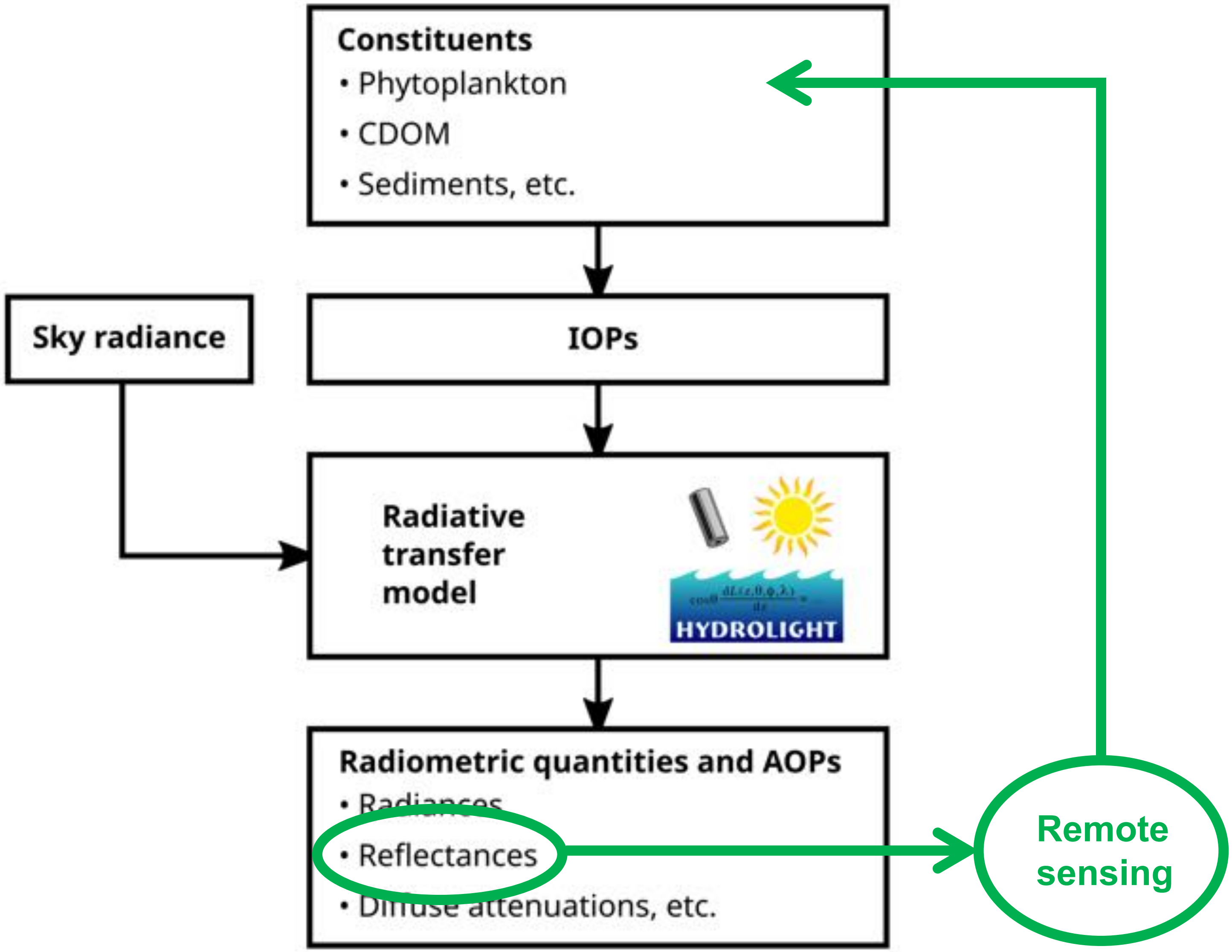
Fieldwork, some years ago...

Coral Reef Targeted Research & Capacity Building for Management

Non-professional activities

I live between two National Parks, Exmoor and Dartmoor!

Forward Modelling



The role of radiative transfer modelling

Understanding how IOPs relate to AOPs

- Characterise relationship between constituents and reflectance

Feasibility studies

- What might be detectable in the reflectance?
- Sensitivity analyses
- Consequence of variability on Rrs

Develop inversion algorithms

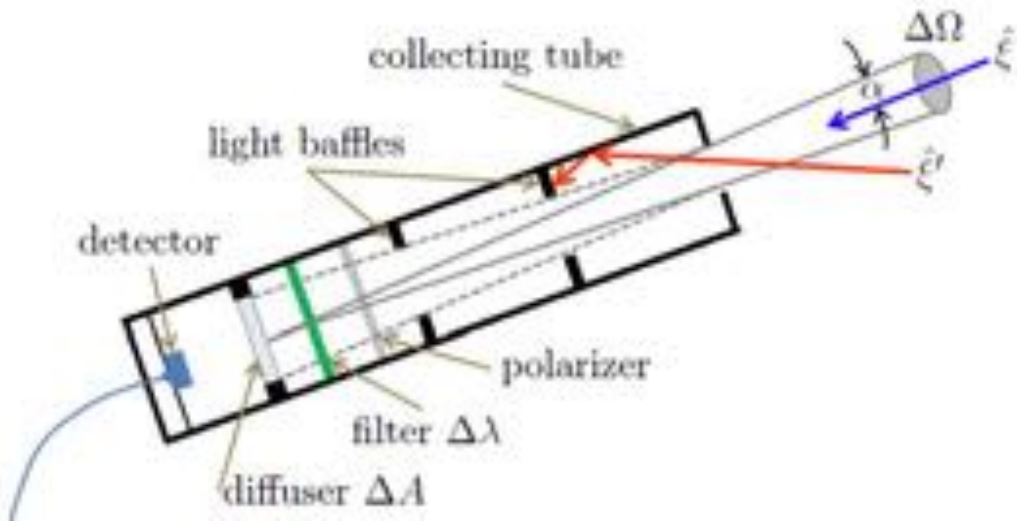
- Look-up tables
- Components of semi-analytical algorithms by regression e.g. convert $r_{rs}(\lambda)$ to $R_{rs}(\lambda)$
- Train machine learning

Learning tool

- Build an intuitive understanding of Hydrological optics

Radiance, ***L*(λ),** units W m^{-2} sr^{-1} nm^{-1}

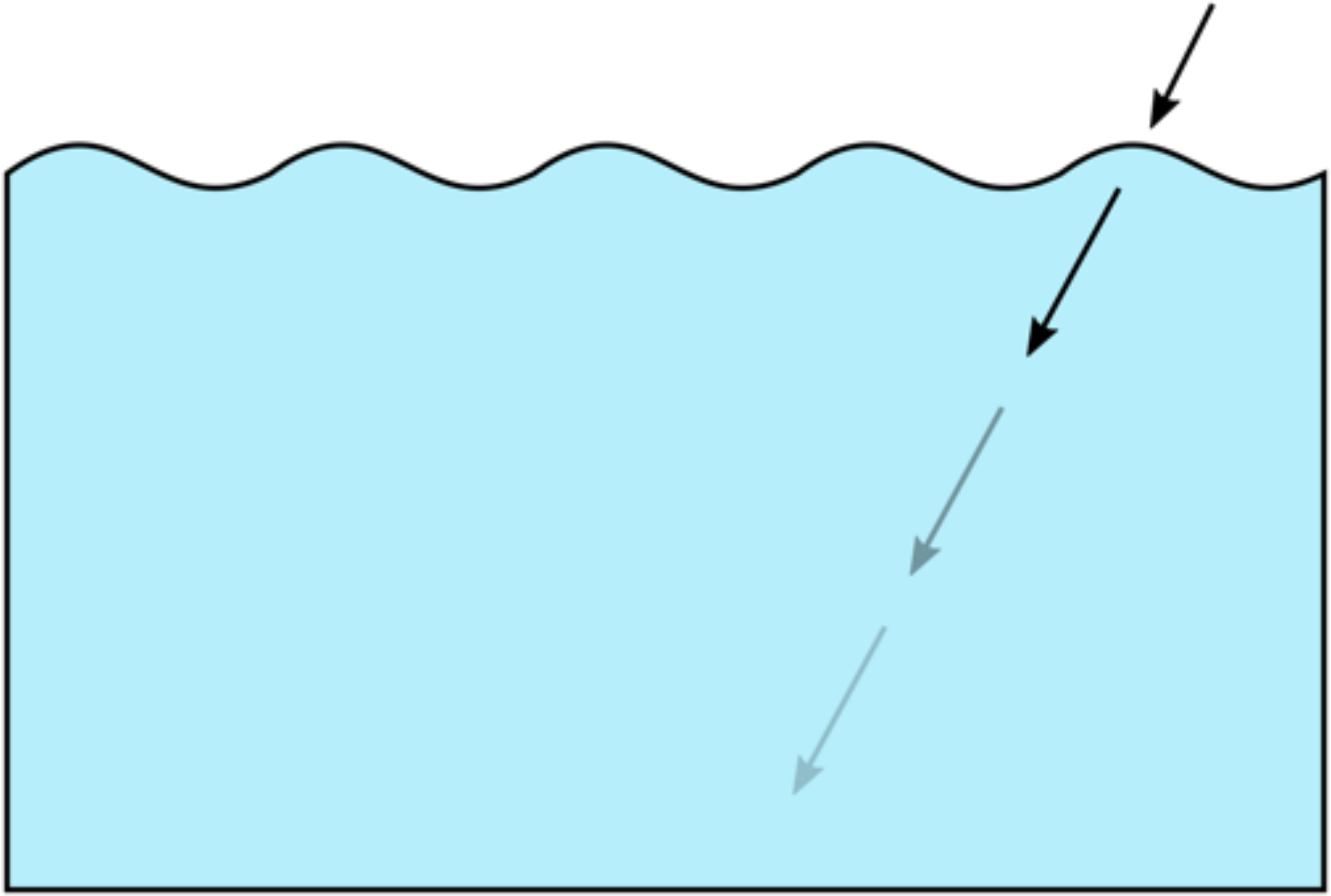
- Spectral radiance is the fundamental radiometric quantity of interest.
- All other radiometric quantities can be derived from it (irradiances, etc).



(source, Ocean Optics WebBook)

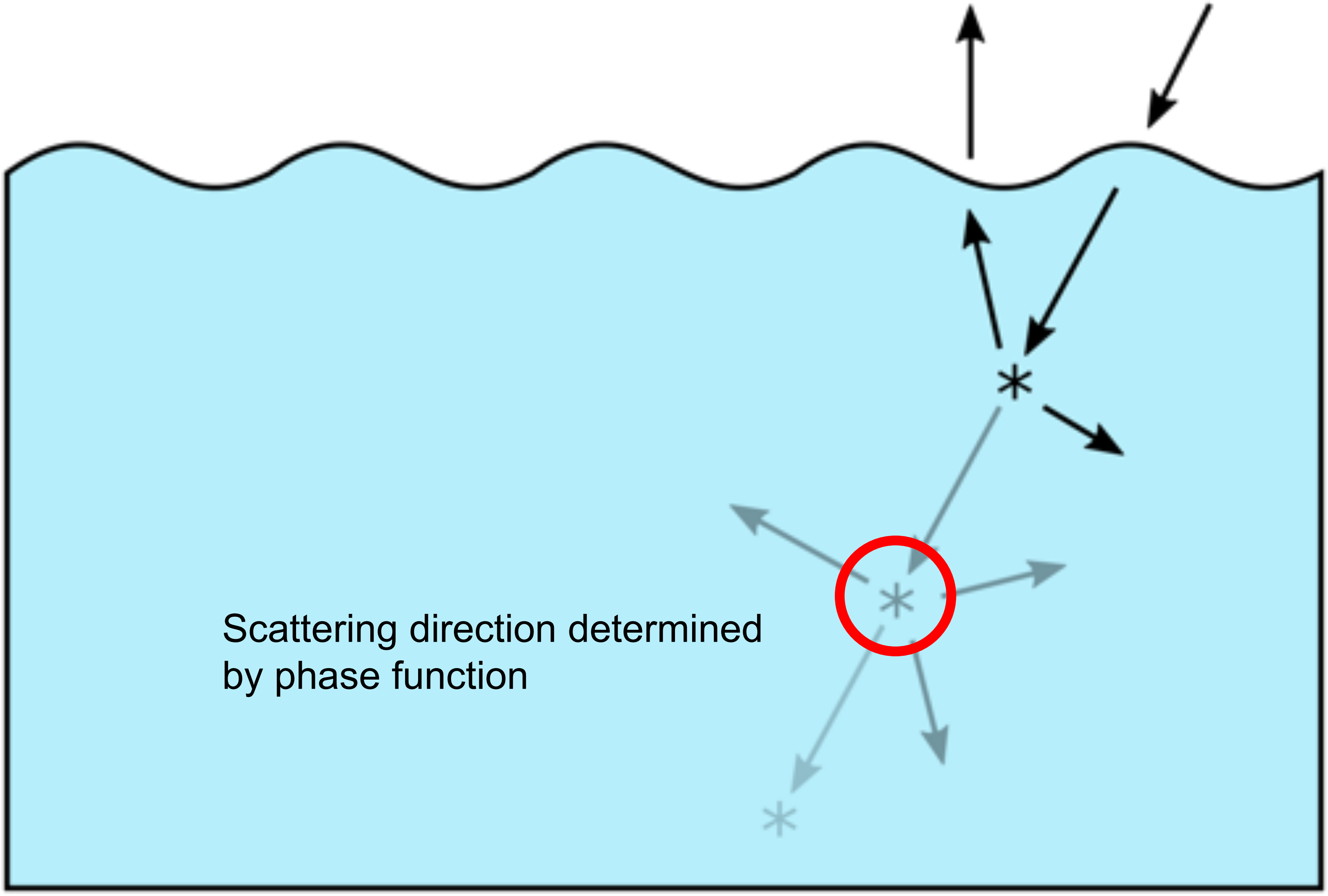
- ➢ **Scalar radiative transfer** (neglects polarisation)
- ➢ **Vectorial radiative transfer** (includes polarisation)

The problem to be solved



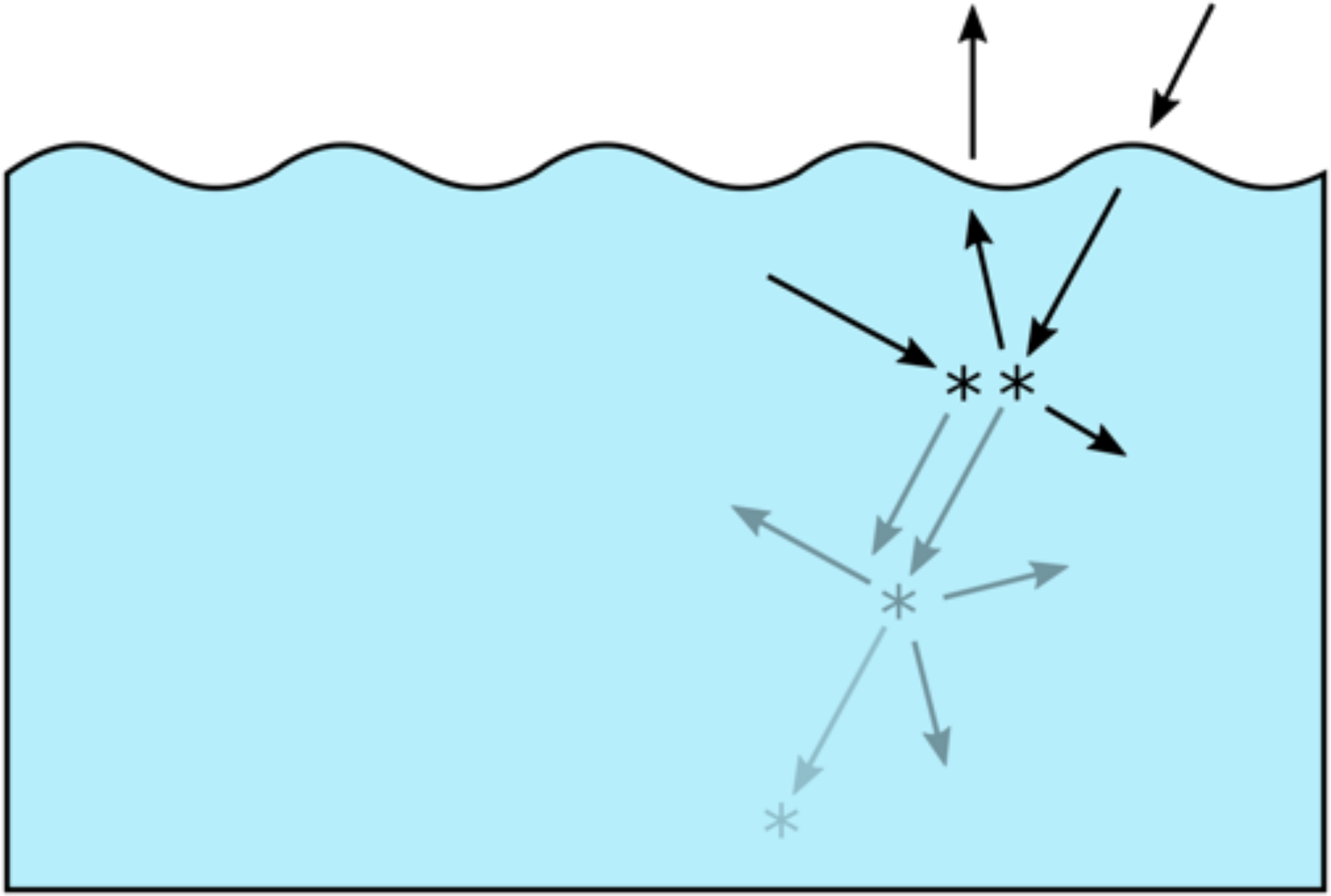
Example 1: No scattering, only absorption

Single scattering - losses to a beam



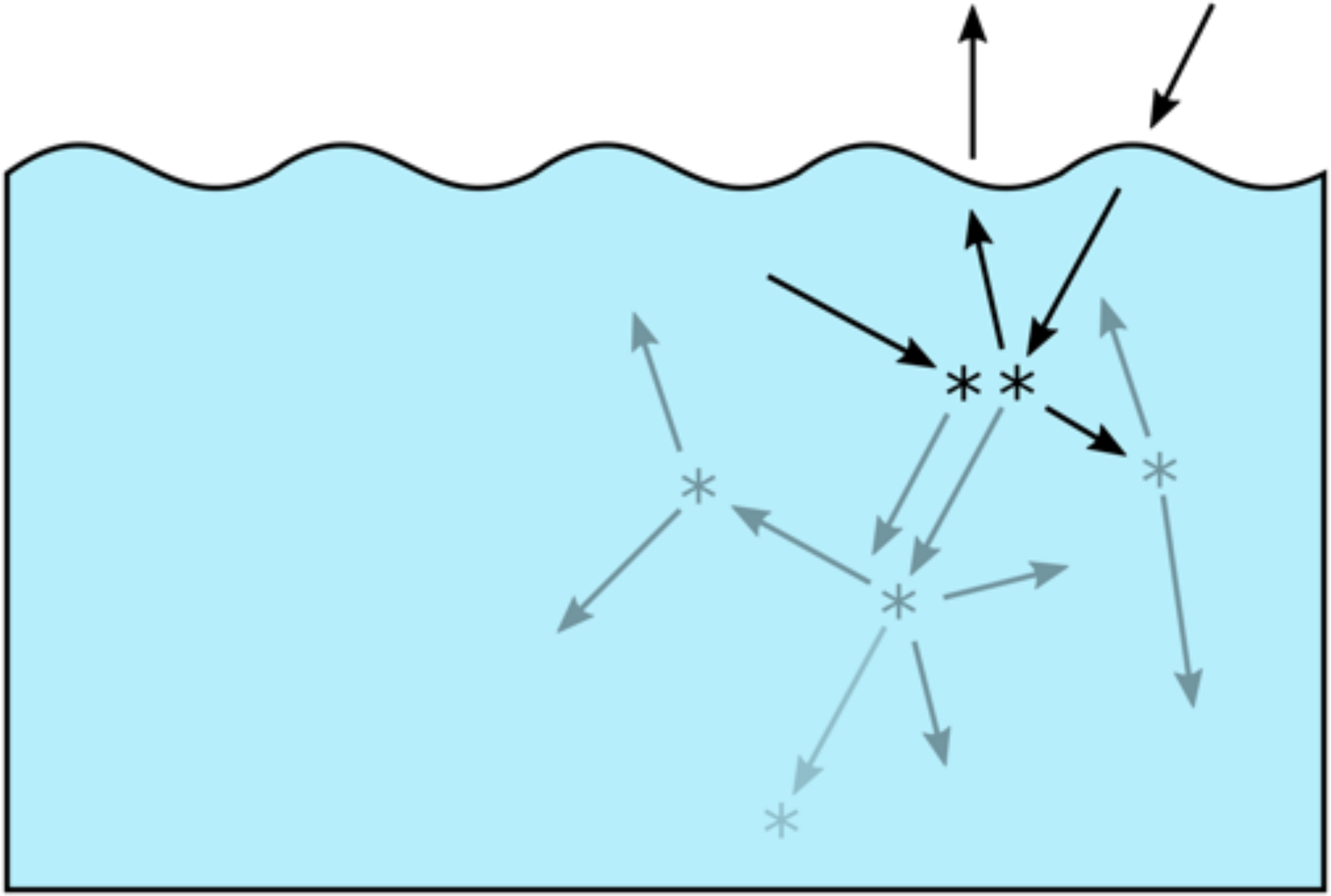
Losses due to absorption + scattering = beam attenuation (a + b = c)

In-scattering



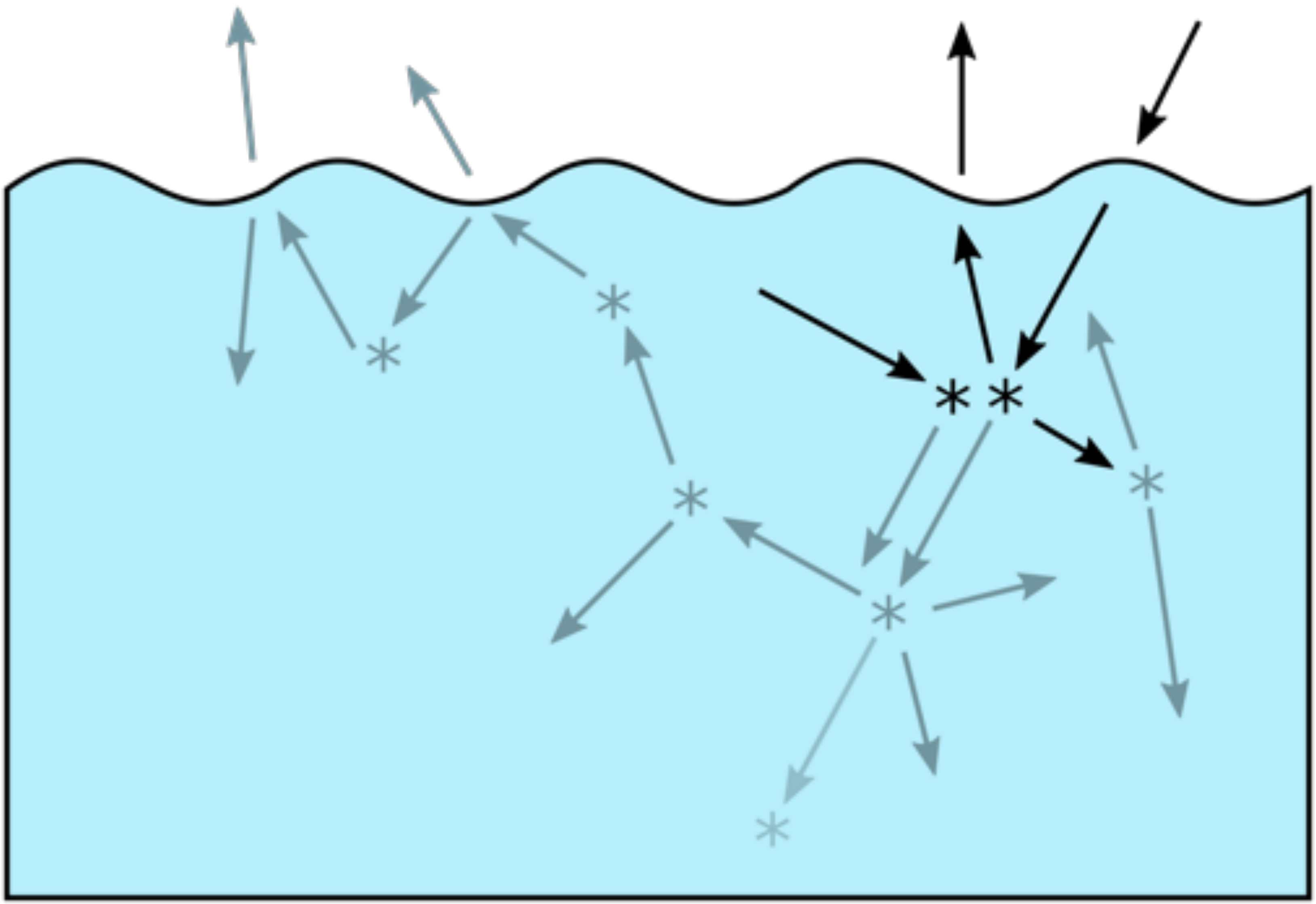
“In-scattering” of light from other directions

Multiple scattering



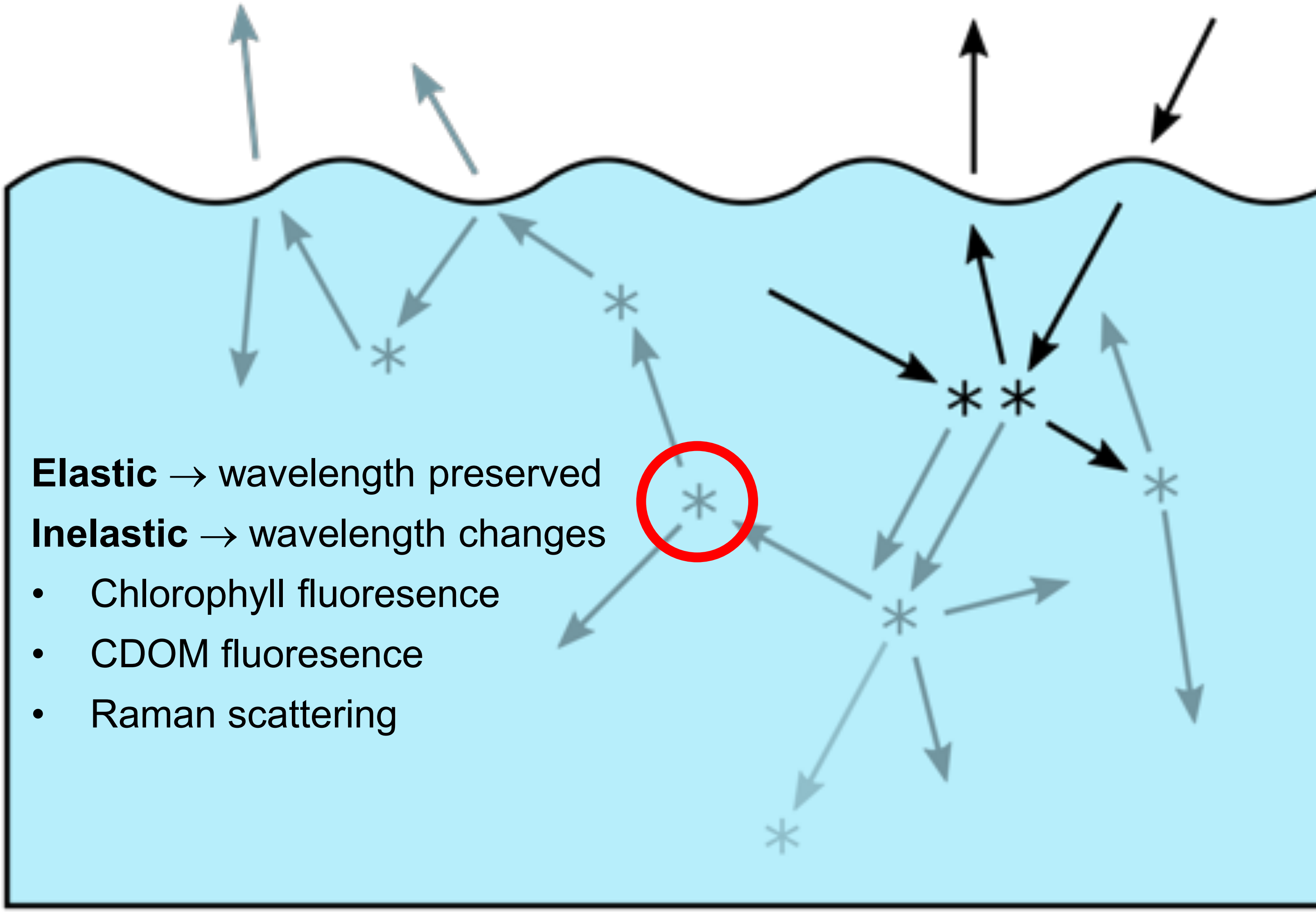
Two orders of scattering in water

Multiple scattering including air-water interface



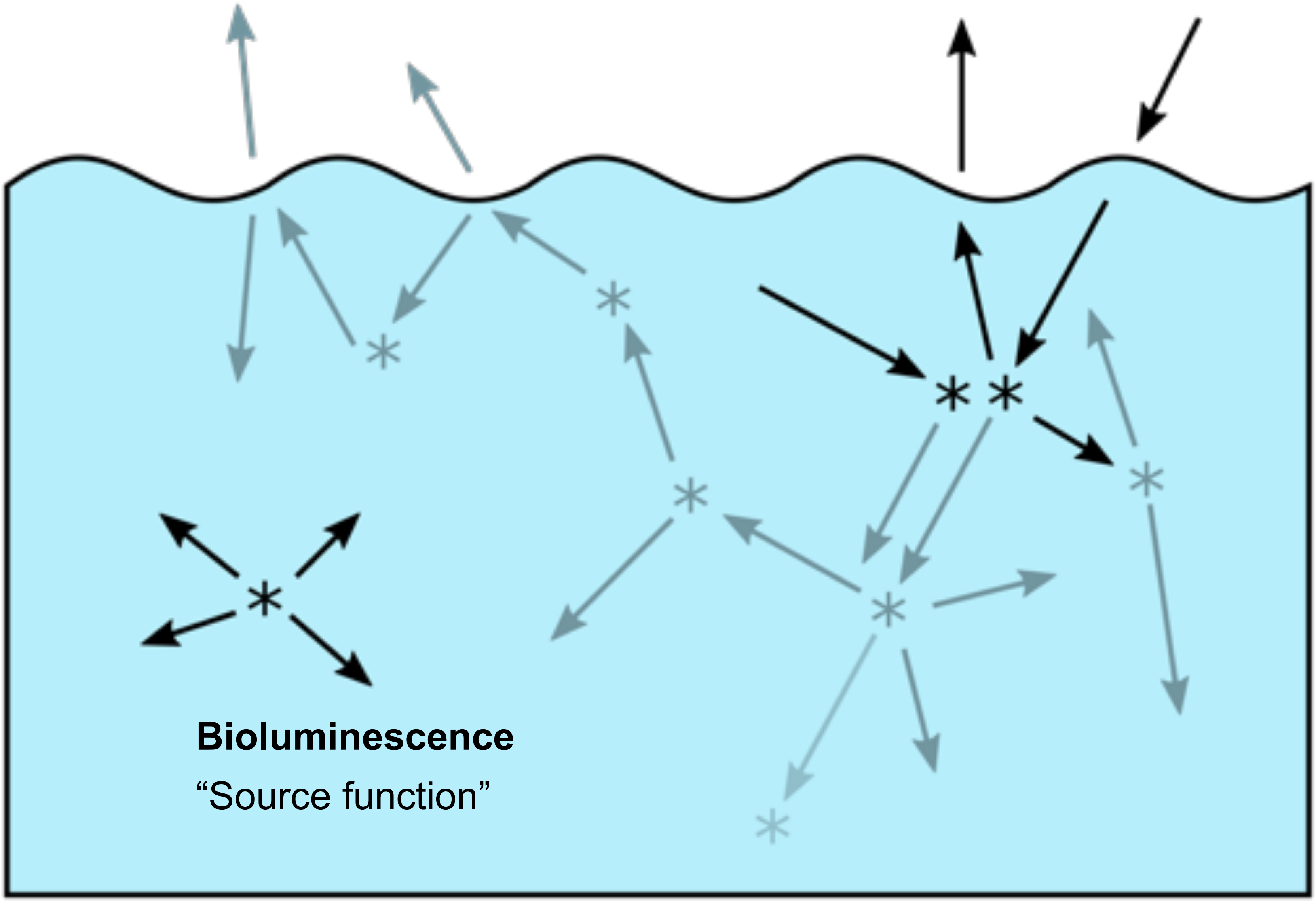
Multiple orders of scattering including from the water surface underside

Inelastic scattering (fluoresence) vs. elastic scattering



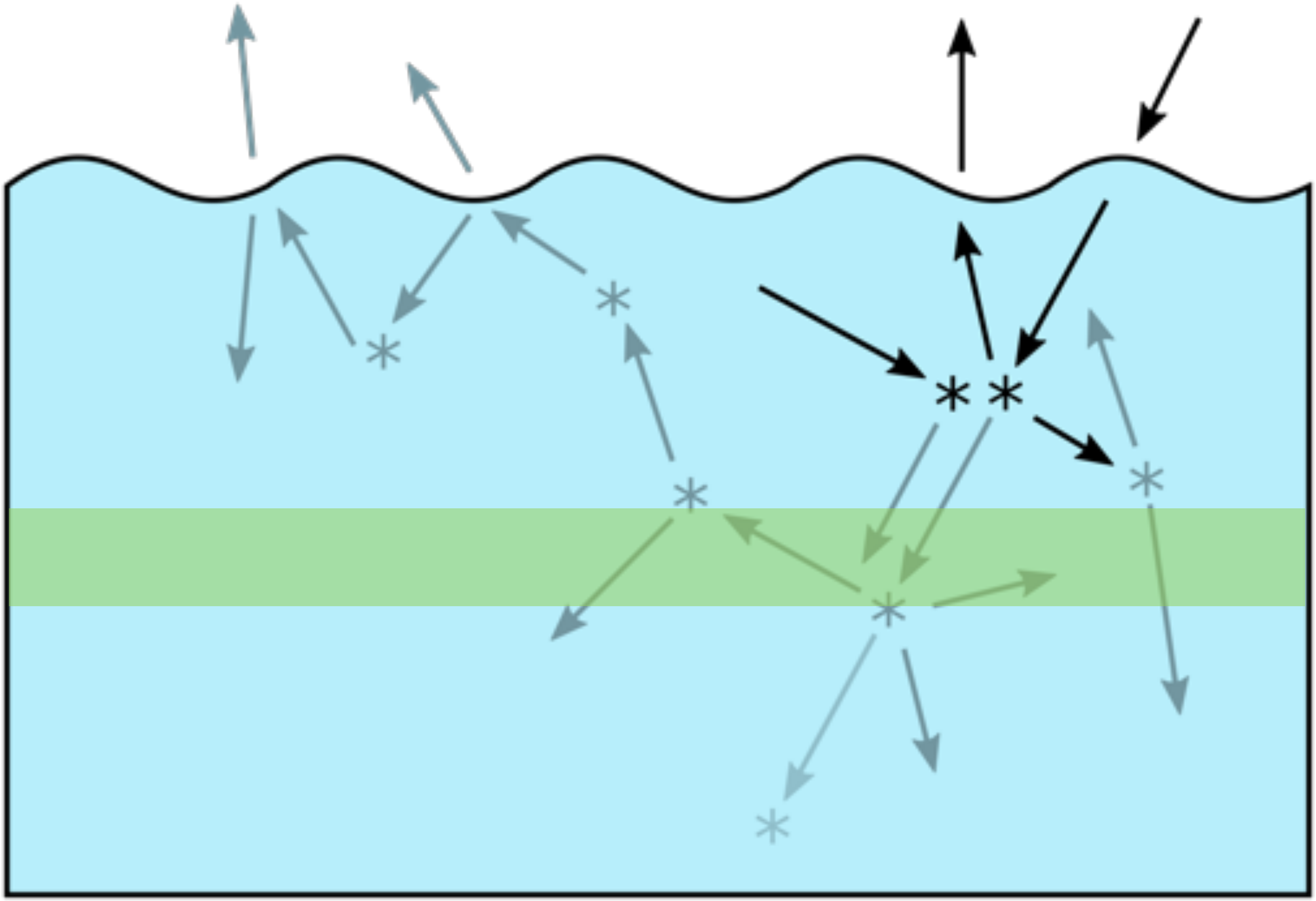
Typically wavelength gets longer (loss of energy) (but not always!)

Bioluminescence



A source of light within the system

Vertical structure



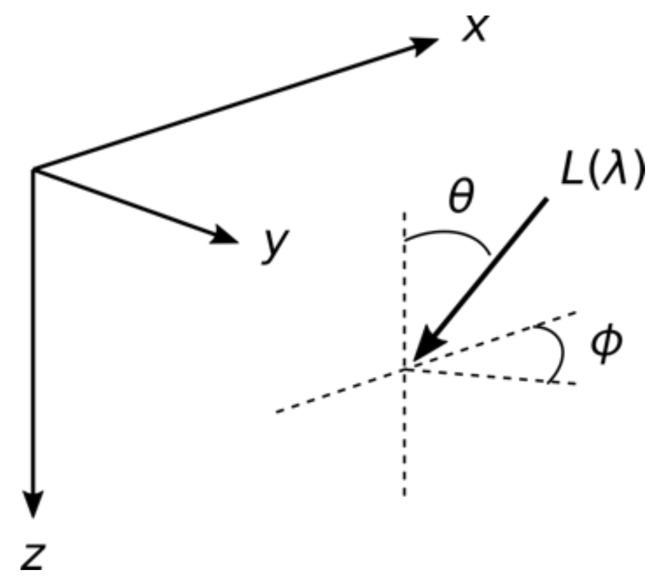
Example – a layer at depth, some paths interact with it some do not

The Complete Solution

Would be:

The radiance distribution (*L*)

- In every direction
- At every point in space
- For each wavelength



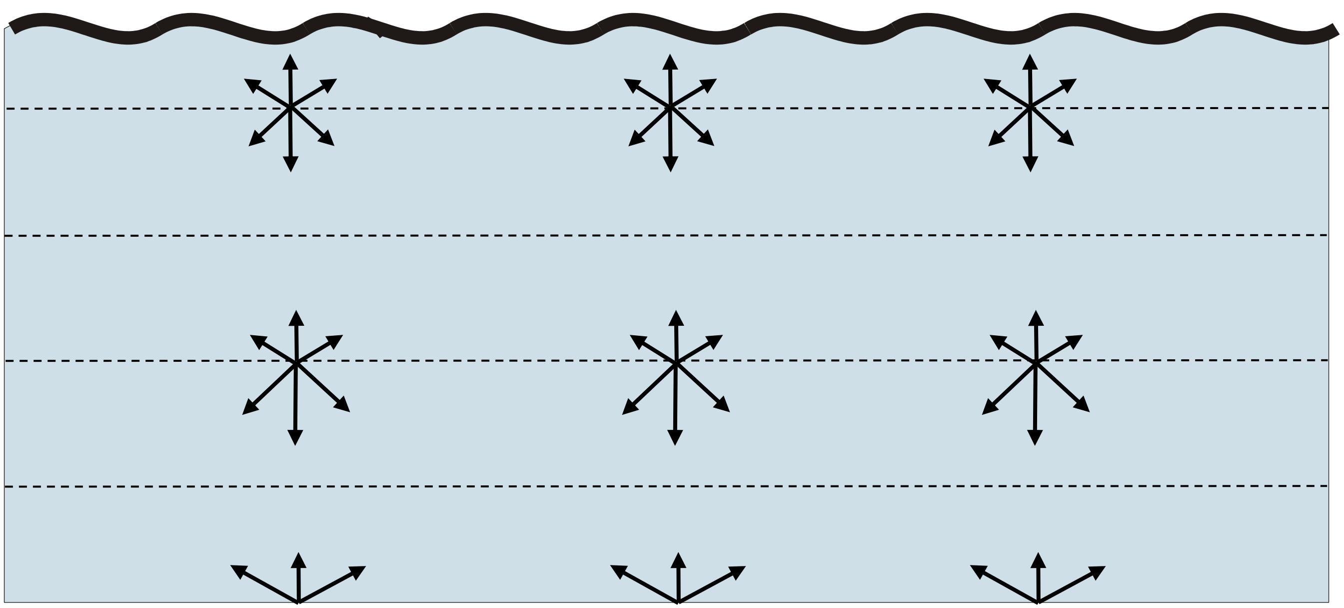
I.e.

$$L(x, y, z, \theta, \phi, \lambda) \qquad (\mathrm{Wm^{-2}sr^{-1}nm^{-1}})$$

From the complete radiance distribution every other radiometric quantity or property can be derived, irradiances, reflectances, diffuse attenuation coefficients (*K* values), etc.

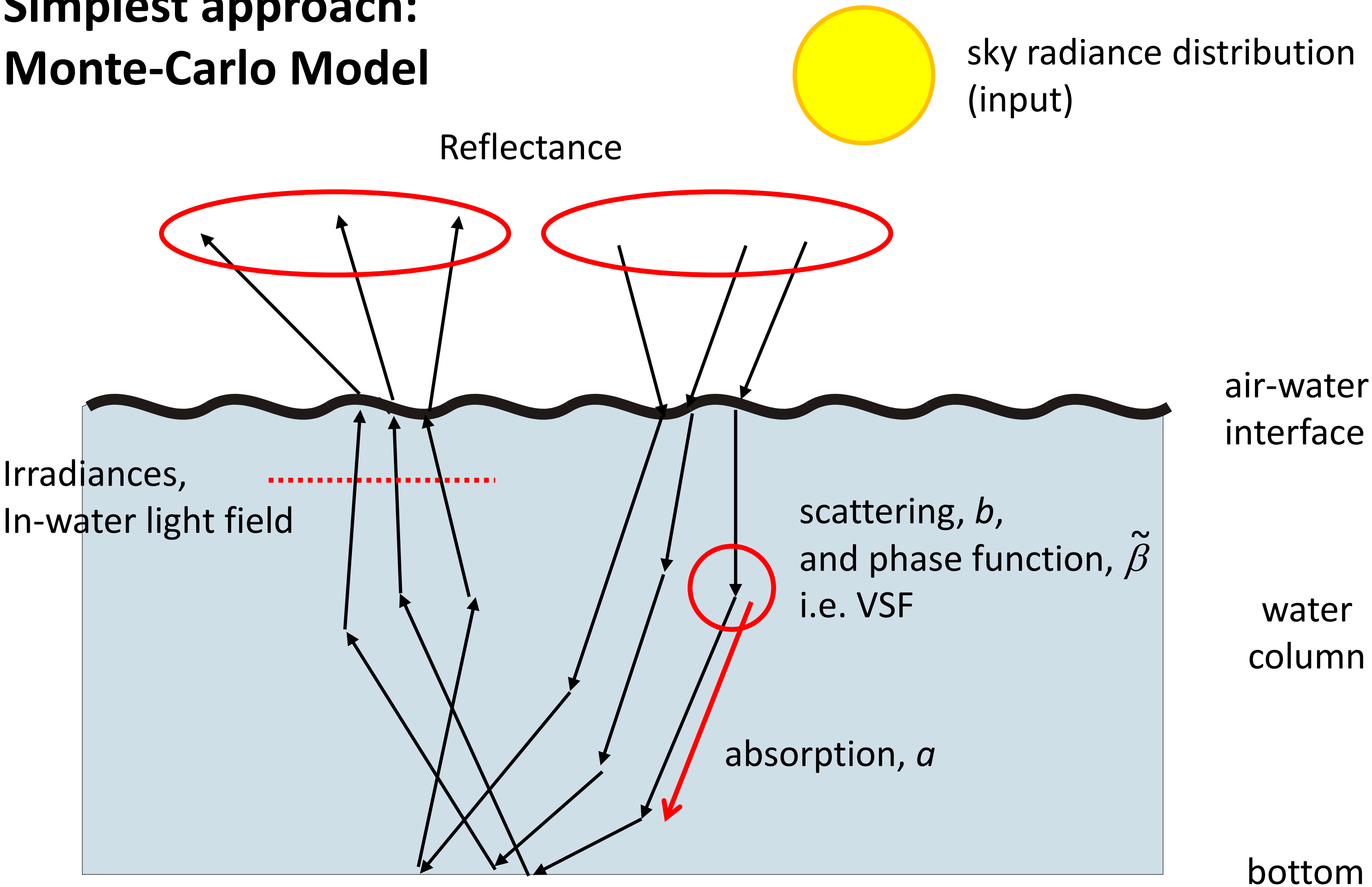
More efficient solution - First step, plane parallel model

- Assume radiance distribution is the same across horizontal planes
- It does not depend on x and y (horizontal position)
- 3D problem becomes 1D
- Very reasonable approximation for deep water or homogenous bottoms



so now we want to determine this $L(z, \theta, \phi, \lambda)$ ($Wm^{-2}sr^{-1}nm^{-1}$)

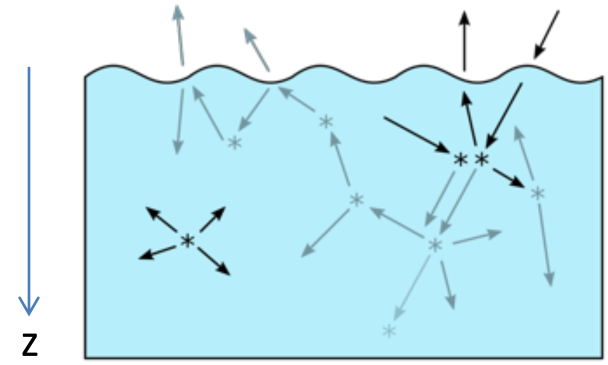
Simplest approach: Monte-Carlo Model



→ Close association between implementation and physical concepts

→ But, **inefficient** and subject to **statistical noise**

Radiative Transfer Equation (RTE)



Change in radiance due to scattering and absorption when moving in +z can be captured by an equation.

One-dimensional, time independent, scalar RTE, no inelastic scattering

$$\cos\theta \frac{dL(z,\theta,\phi,\lambda)}{dz} = -[a(z,\lambda) + b(z,\lambda)]\, L(z,\theta,\phi,\lambda) + b(z,\lambda) \int_0^{2\pi}\int_0^{\pi} L(z,\theta',\phi',\lambda)\, \tilde{\beta}(z,\theta',\phi' \rightarrow \theta,\phi,\lambda) \sin\theta'\, d\theta'\, d\phi' + S(z,\theta,\phi,\lambda)$$

attenuation

in-scattering

additional sources

What we want to know is: $L(z, \theta, \phi, \lambda)$ (as underlined in red)

This describes how the full directional radiance distribution changes as you take a small step down through the water column (i.e. *z* increases).

HydroLight

→ Software model that solves the RTE

- Developed by Curt Mobley working with Rudy Preisendorfer, starting in 1978.
- Commercial product on PC since 1998.
- Over 200 users in 30 countries and used in many publications.
- Google scholar search for “HydroLight” returns > 2600 results.
- As of 2017 ownership of HydroLight passed to me (John Hedley) and is now a product of Numerical Optics Ltd.
- Commercial basis has always been the only support for maintenance and development of the software.
- Latest version is version 6.0, now also available for Mac and Linux.

See the document *HydroLight_History.pdf* for more historical info.

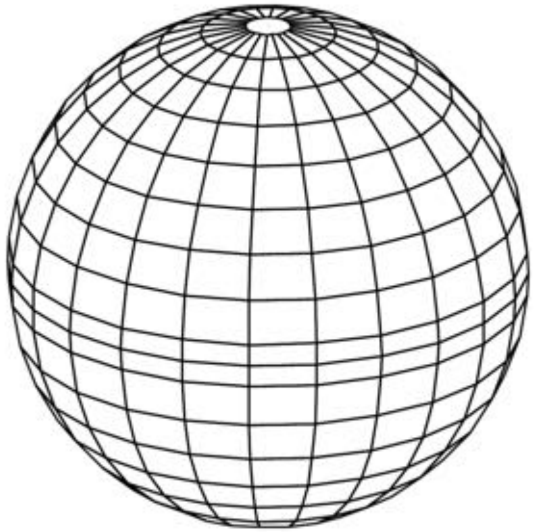
Discretisation of direction

HydroLight standard discretisation

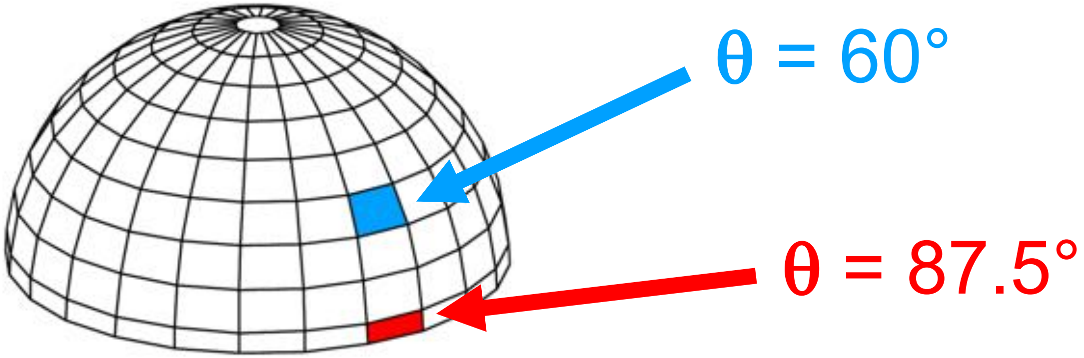
Resolution is $10° \times 15°$

Full sphere of directions 18 x 24 quads
plus end-caps = 434 entries

Work with quad averaged radiances



Or consider separate hemispheres
E.g. downwelling
quad averaged radiance

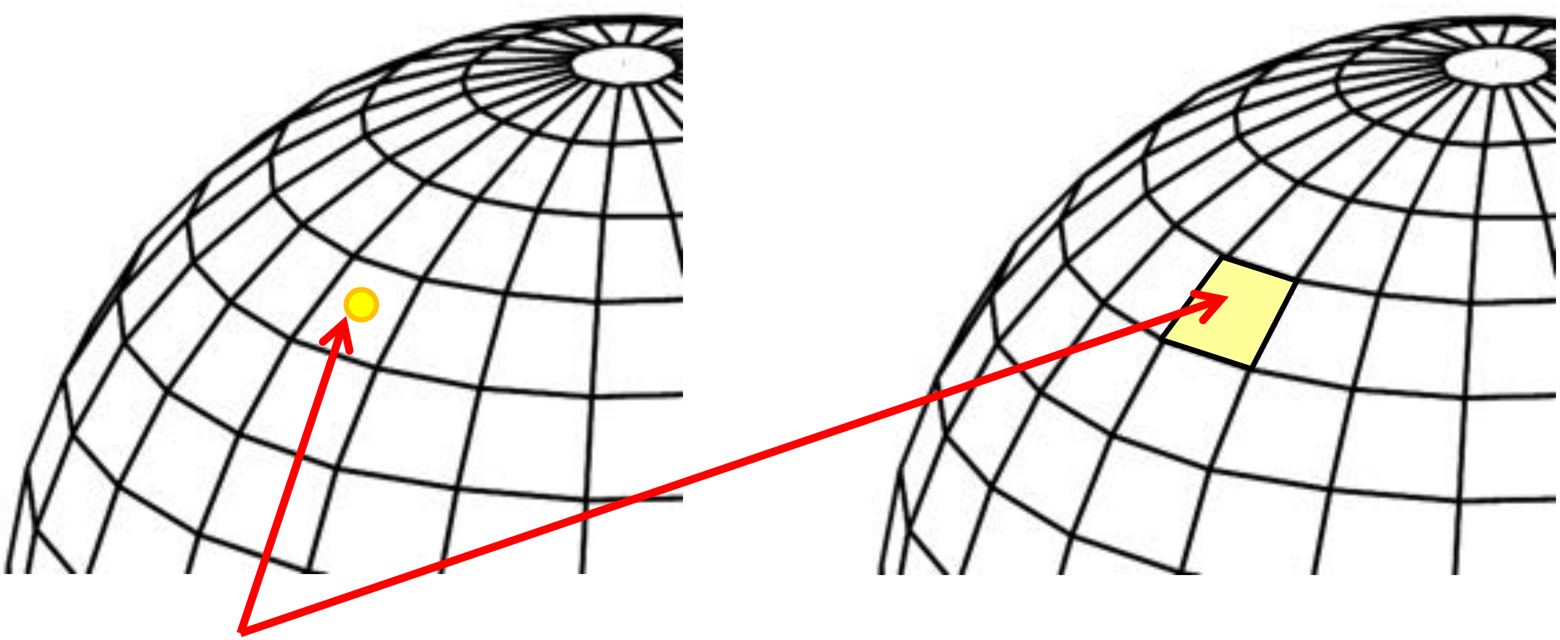


$L(z, \theta, \phi, \lambda)$ $\Rightarrow$ a table of 434 numbers for any particular z and λ

Quad-averaged radiances

The solar disc is smaller than one quad

So one consequence is that the direct solar radiance is spread over the quad



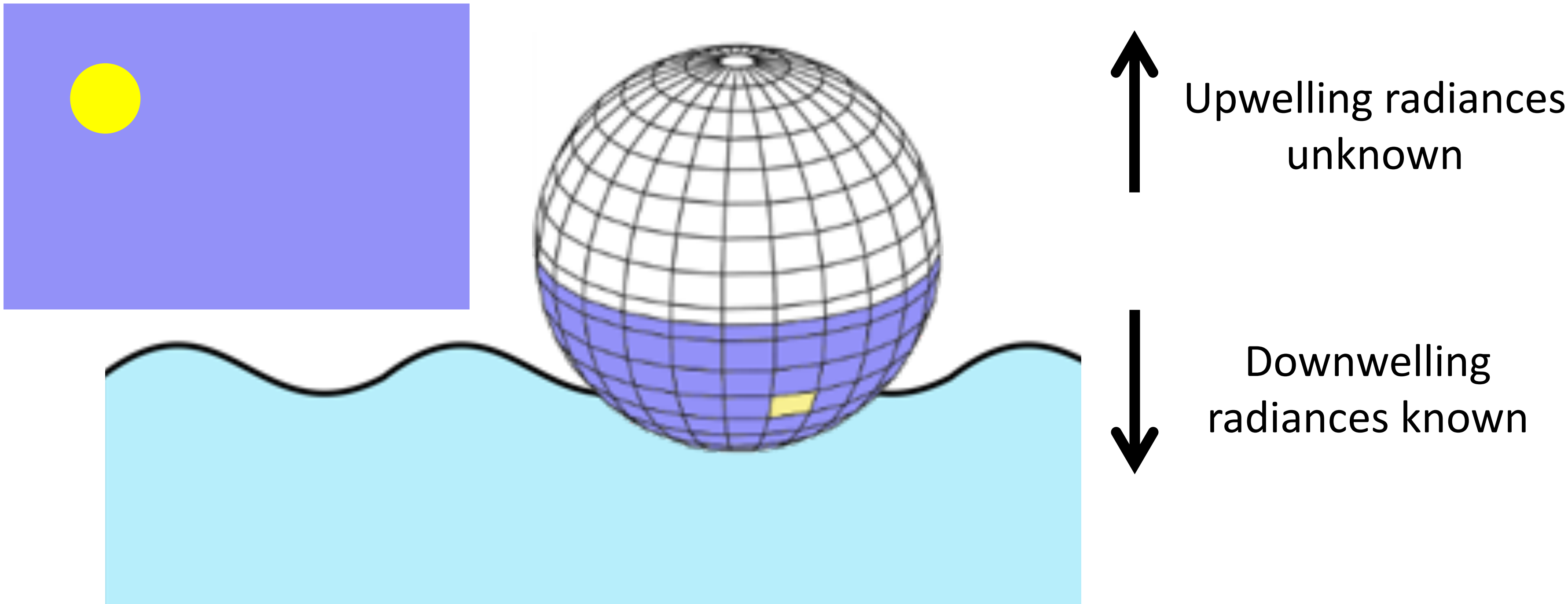
However the total energy as averaged over the quad is the same in both cases and correct.

Makes almost no difference to most quantities of interest, due to scattering the direct radiance is rapidly spread out underwater anyway.

See Tech Note: HTN2_AngularResolution.pdf

Sky Radiance Distribution

Is an input, considered known, can be supplied or HydroLight has a built-in model.



Reason why solving the RTE is non trivial is that at the start we only know the downwelling radiances at the top of the water column.

The other information we need is at the bottom boundary - either the bottom reflectance or the assumption of infinite depth.

Solution – “invariant imbedded method”

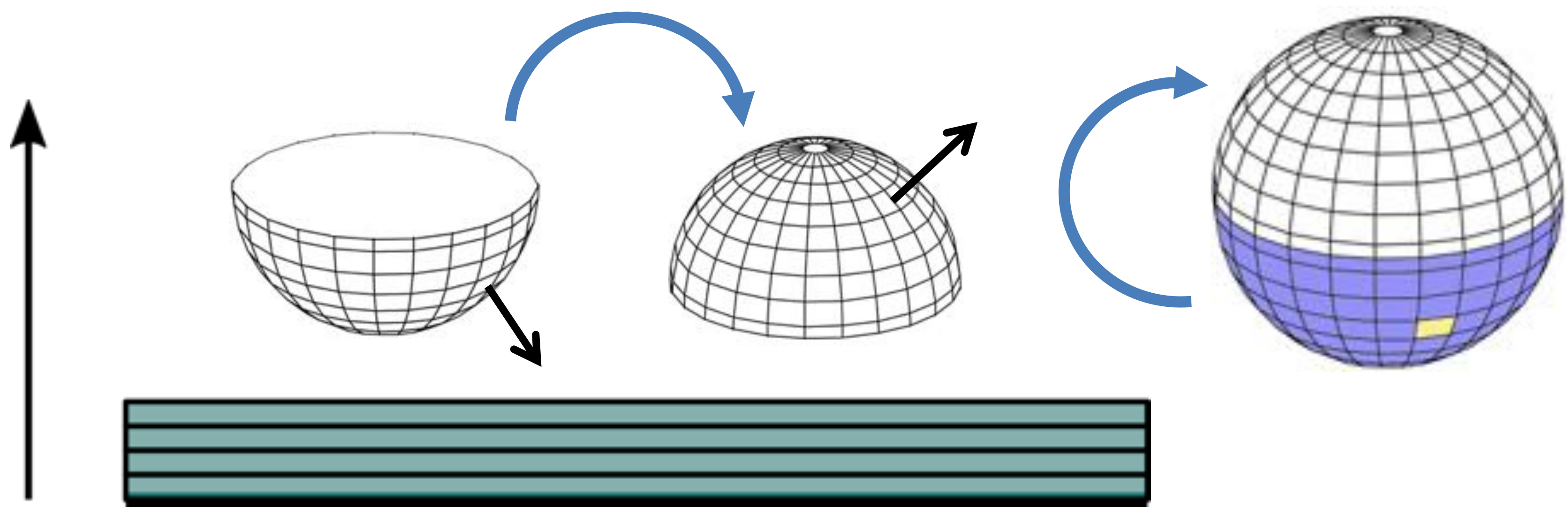
Techniques HydroLight uses to solve the RTE

1. The **invariant imbedded method**
   – in essence, reflectance is propagated to the top
   – very large matrix of numbers, 434 × 434

Add thin layers of water from the bottom up

Reflectance gives upward radiance from downward

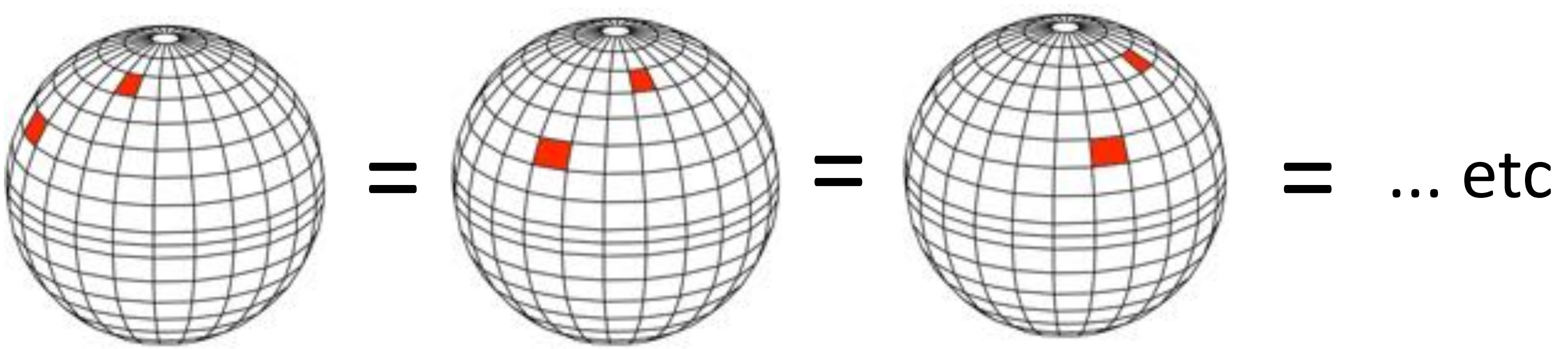
Can then calculate upward radiances at the top



Exploit symmetries

2. Exploit the fact that the **phase function is dependent on relative angle only**.

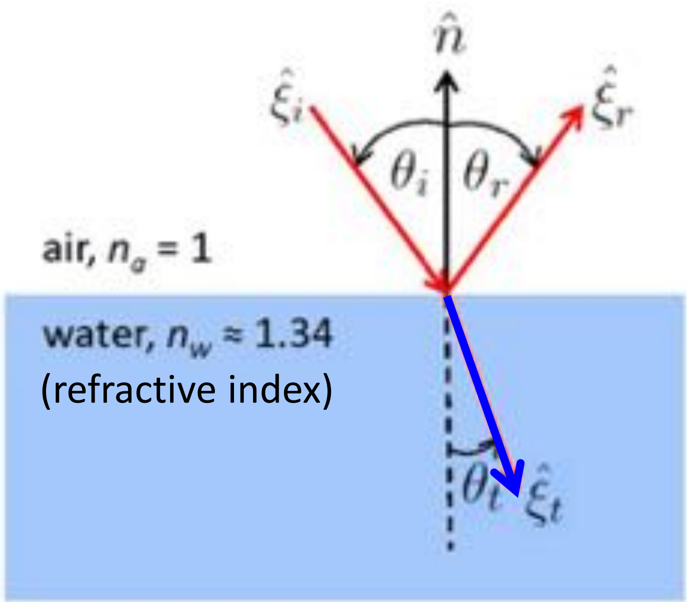
The matrices can be transformed into matrices with lots of zeros in them – more computationally efficient.



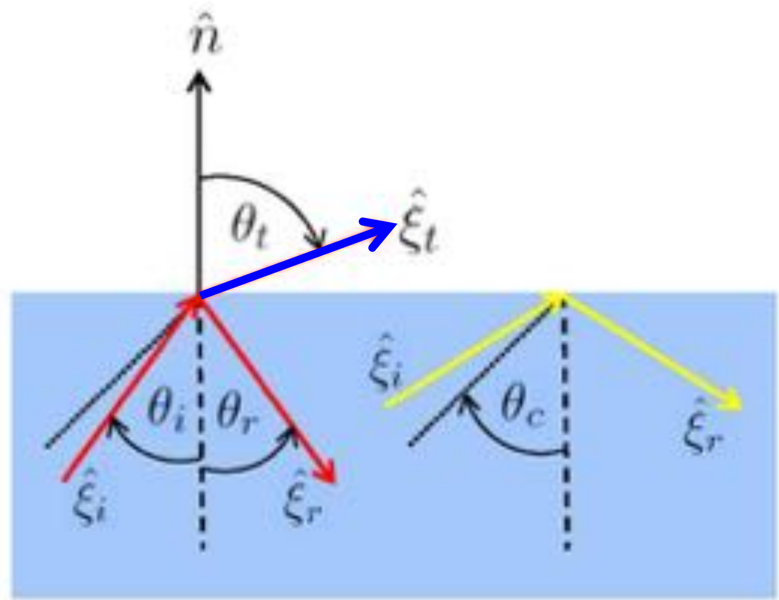
See the Ocean Optics Webbook or Mobley's *Light and Water* (1994) for details.

Air-water interface – flat surface   (Fresnel equations)

air incident

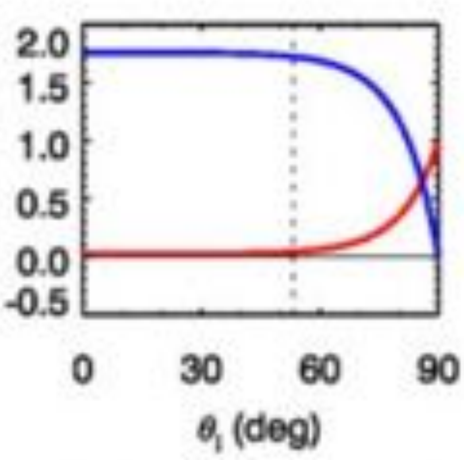


water incident



transmission

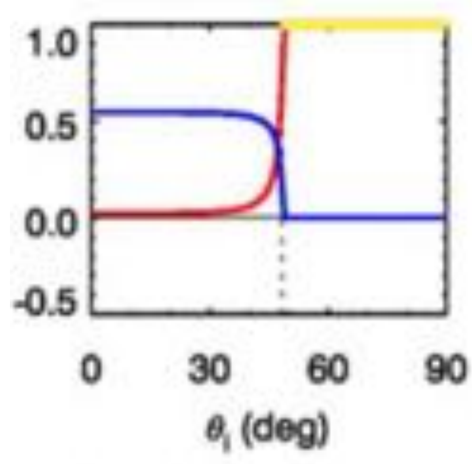
reflectance



transmission

reflectance

total internal reflectance

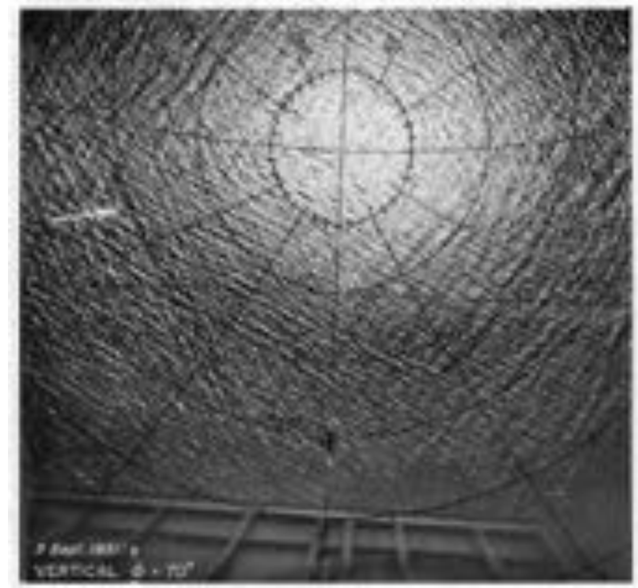


Cox-Munk wind-speed wave-slope law

Cox and Munk equations

- 1950s - based on photographs of surface glitter
- Many subsequent studies: all agree

Cox & Munk (1956) Slopes of the Sea Surface Deduced from Photographs of Sun Glitter. *Scripps Inst. Oceanogr. Bull*. 6(9): 401–88



Result is statistical model of the slopes of the sea surface:

U_{10} = wind speed ms^{-1}

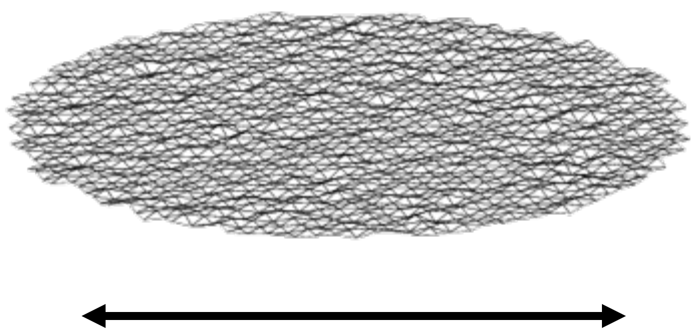
Mean square slope (along wind) $\sigma_w^2 = 0.00316\, U_{10}$

Mean square slope (cross wind) $\sigma_c^2 = 0.00192\, U_{10}$

- Predicts bigger slopes in the wind direction (wave peaks and troughs).
- Says nothing about wave heights or the coherent structure of the surface.
- Works only at large scales and open seas (mature seas).

Air water interface modelling

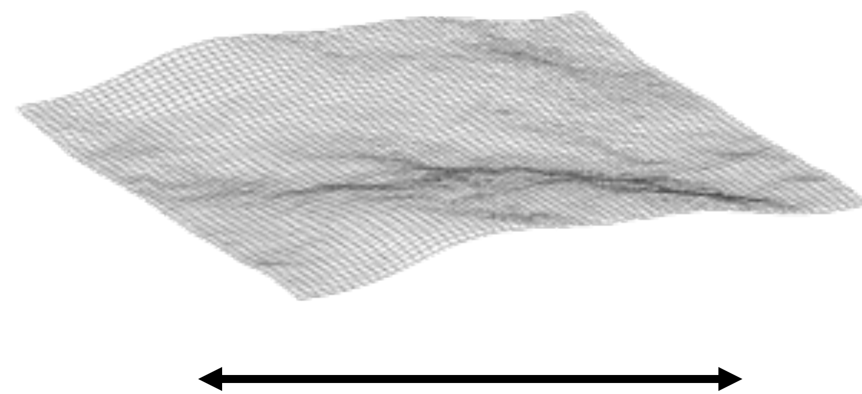
Cox-Munk slope statistics only



scale invariant

No wave structures

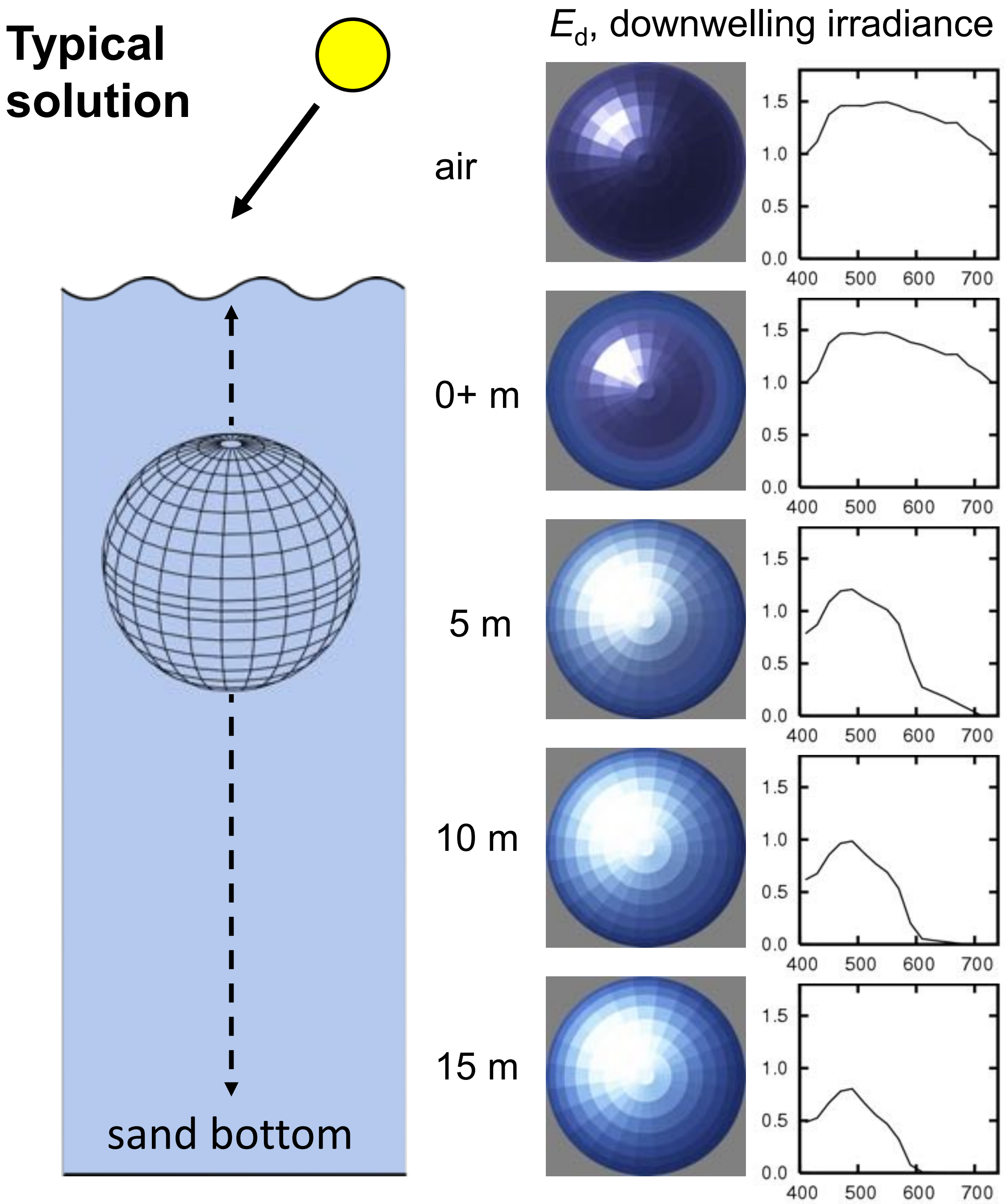
Slope and elevation statistics

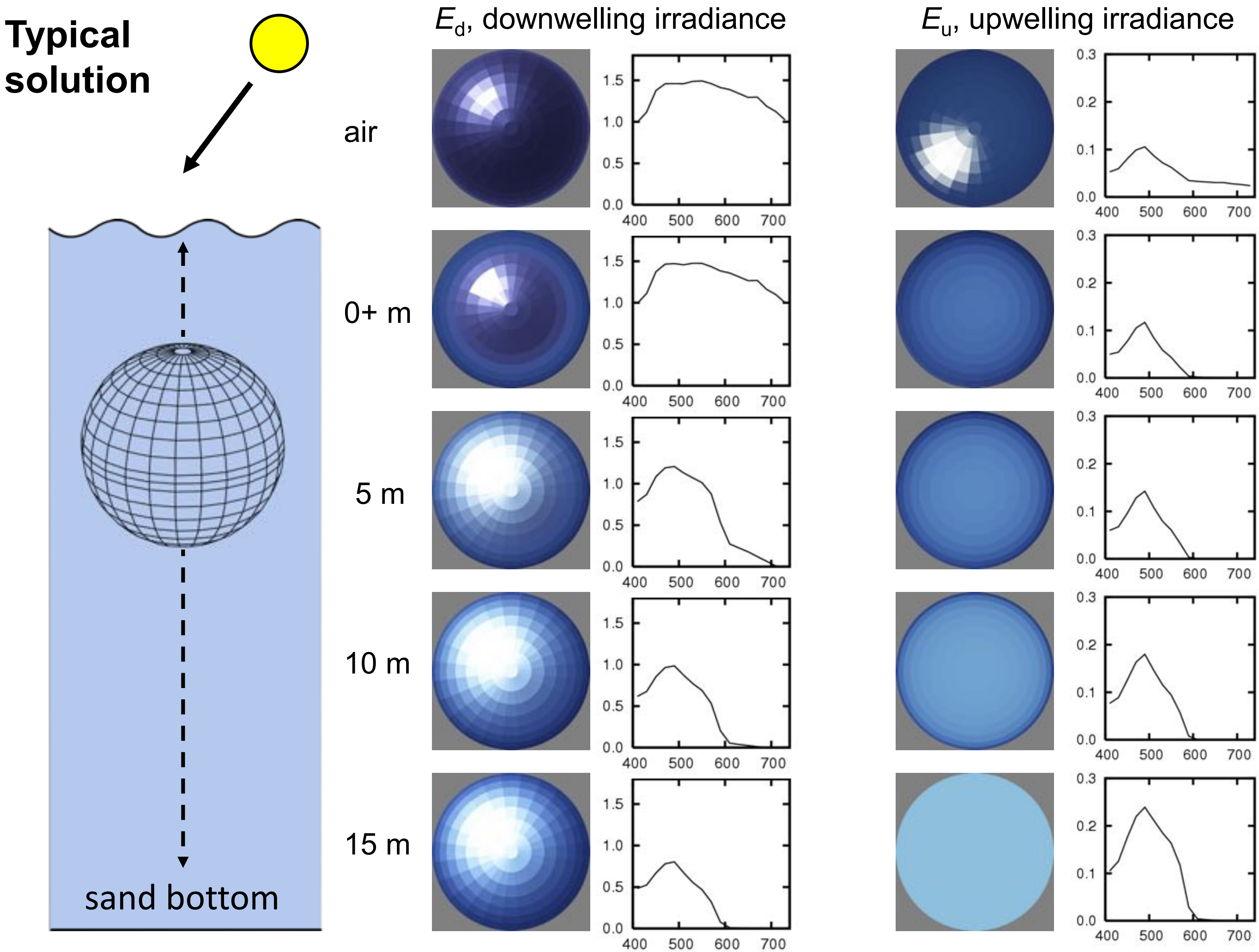


3 m

Needs to be large and detailed enough to cover features from 100s m to millimetres

- Ray tracing is used to characterise the directional reflectance and transmittance - pre-calculated functions.


Typical solution
E_d, downwelling irradiance
air
0+ m
5 m
10 m
15 m
sand bottom
1.5
1.0
0.5
0.0
400
500
600
700


Typical solution
air
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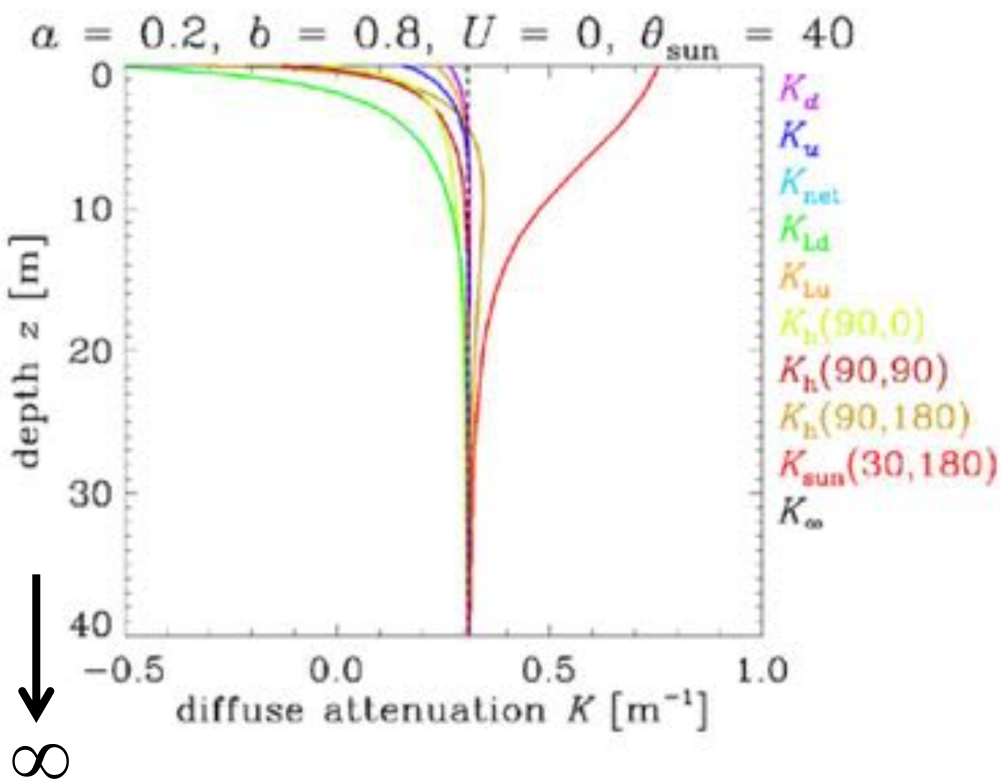
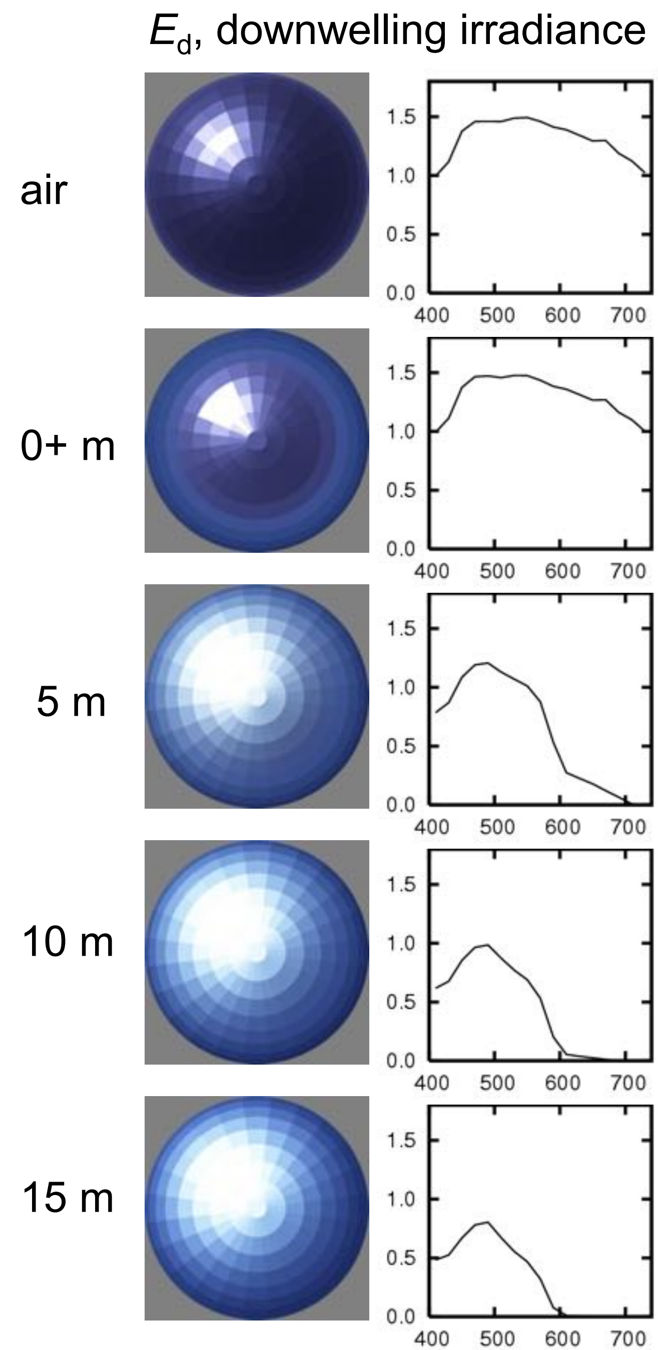
E_d, downwelling irradiance
E_u, upwelling irradiance
400
500
600
700
0.0
0.5
1.0
1.5
0.1
0.2
0.3

Infinite depth (homogenous IOPs)





Tends to a constant relative directional distribution of light, azimuthally constant

→ **Asymptotic radiance distribution**

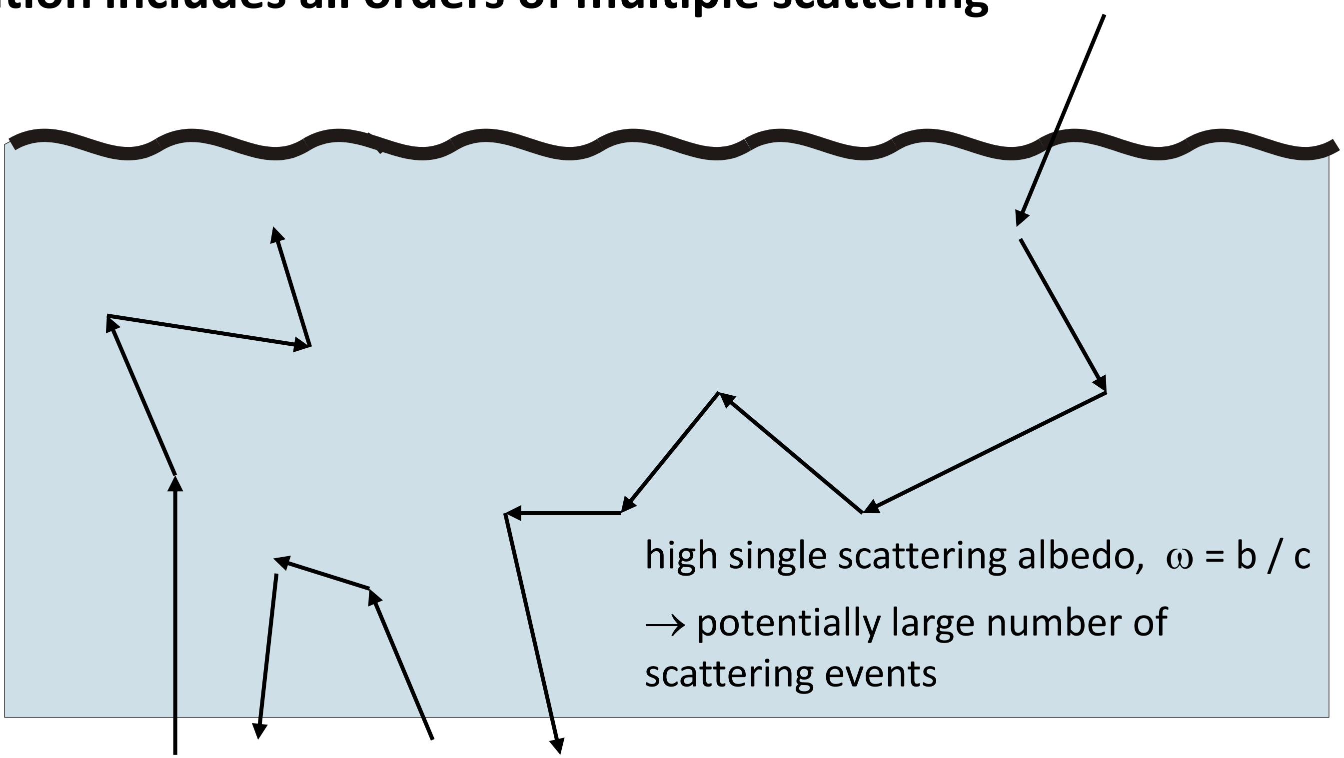
Amount of light decreases with depth according to an exponential function

Comparison of HydroLight vs. Monte Carlo

- **Run time linearly proportional to optical depth** (attenuation × physical depth)
  Monte Carlo ∝ exp(optical depth)

- **Run time independent of IOP(*z*) complexity, arbitrary depth resolution**
  not a set of homogeneous layers as used in some methods

- **Solution includes all orders of multiple scattering**



HydroLight summary of features and limitations

- Time independent / time averaged
- One spatial dimension (depth) - no restrictions on depth dependence of IOPs (not a “layered” model)
- No restriction on wavelengths, included data from 300 to 1000 nm
- Model for sky radiance onto sea surface, or can load arbitrary data
- Air-water interface model (parameterizes gravity & capillary waves via the wind speed)
- Infinite depth or supplied bottom reflectance are possible options
- Includes all orders of multiple scattering
- Includes Raman scatter by water
- Includes fluorescence by chlorophyll and CDOM
- Includes internal sources (bioluminescent layers)
- Polarization not included (may give errors in computed radiances of up to ~10%, ~1% in irradiances)
- Can run from GUI or from scripts

“Validation” - general discussion

What does it mean?

Probably,

“Comparison of model outputs to empirical data are of acceptable accuracy”

Optical Closure:

E.g. measure IOPs $\rightarrow$ model reflectance $\rightarrow$ compare to satellite data

<u>**Many different aspects that can be “wrong”:**</u>

Physical concepts – plane parallel assumption, scalar approximation

Solution method – e.g. Monte Carlo vs. directionally discretised

Implementation – is the program written correctly, any bugs?

Measurement of empirical data – uncertainties in empirical data

Where is HydroLight on these aspects?

Physical concepts

– physical concepts well accepted within the scope of the model definition
– e.g. scope includes plane parallel assumption, scalar approximation

Solution method

– method is an "exact" physical solution, <u>for quad-averaged radiances</u>!
– numerical issues, only in extreme parameterisations

Implementation

– no serious bugs found in quite a while
– benefit of a long time code-base in use by many people
– bugs are still an ever present danger!

Measurement of empirical data

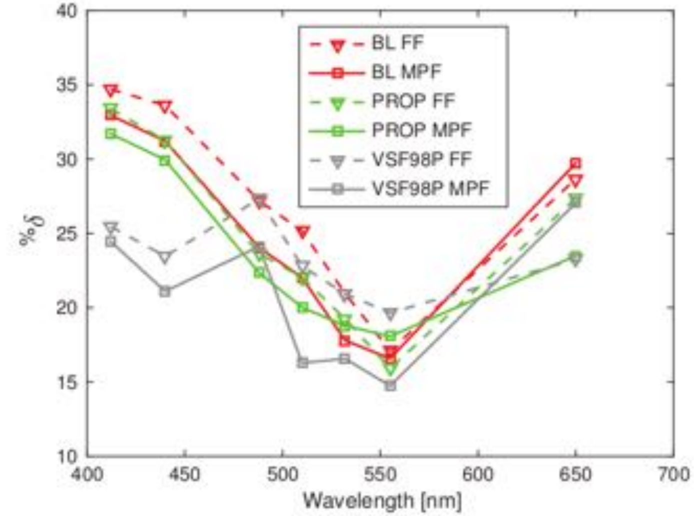
– main area for doubt, both in terms of inputs and output comparisons
– HydroLight includes built-in options, such as phase functions, Chl and CDOM fluoresence, etc. these are empirically based: **USER BEWARE**
– for some real data is scarce, e.g. CDOM fluoresence, only 1 paper!

Examples of optical closure using HydroLight

Tonizzo et al. (2017)
Applied Optics 56, 130-146.

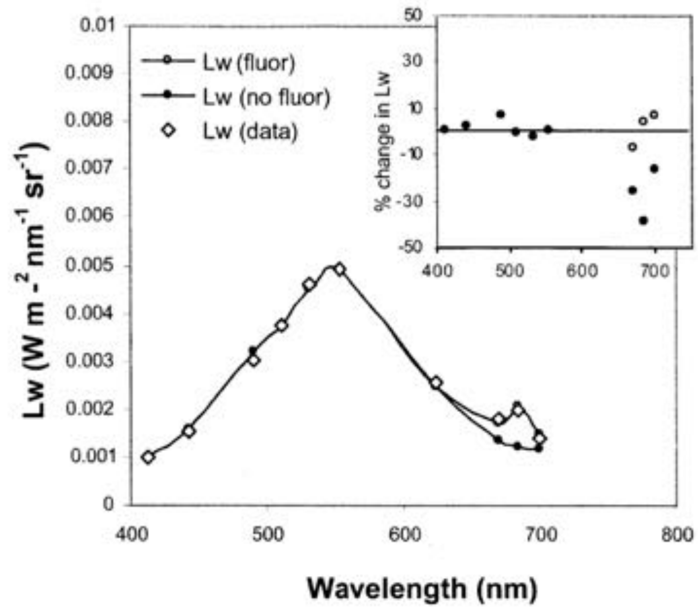
Overal discrepancies between measured R_{rs} and modelled:

- Using measured phase functions ~20%
- Fournier-Forand phase functons ~23%



Tzortziou et al. (2005)
Estuarine, Coastal and Shelf Science 68, 348-362.

Average % difference between modelled and measured water leaving radiances ~7% (0 - 20%)



→ Very careful studies – discrepances of 20% are in general a very good result.

Main Caveat – From the Users' Guide

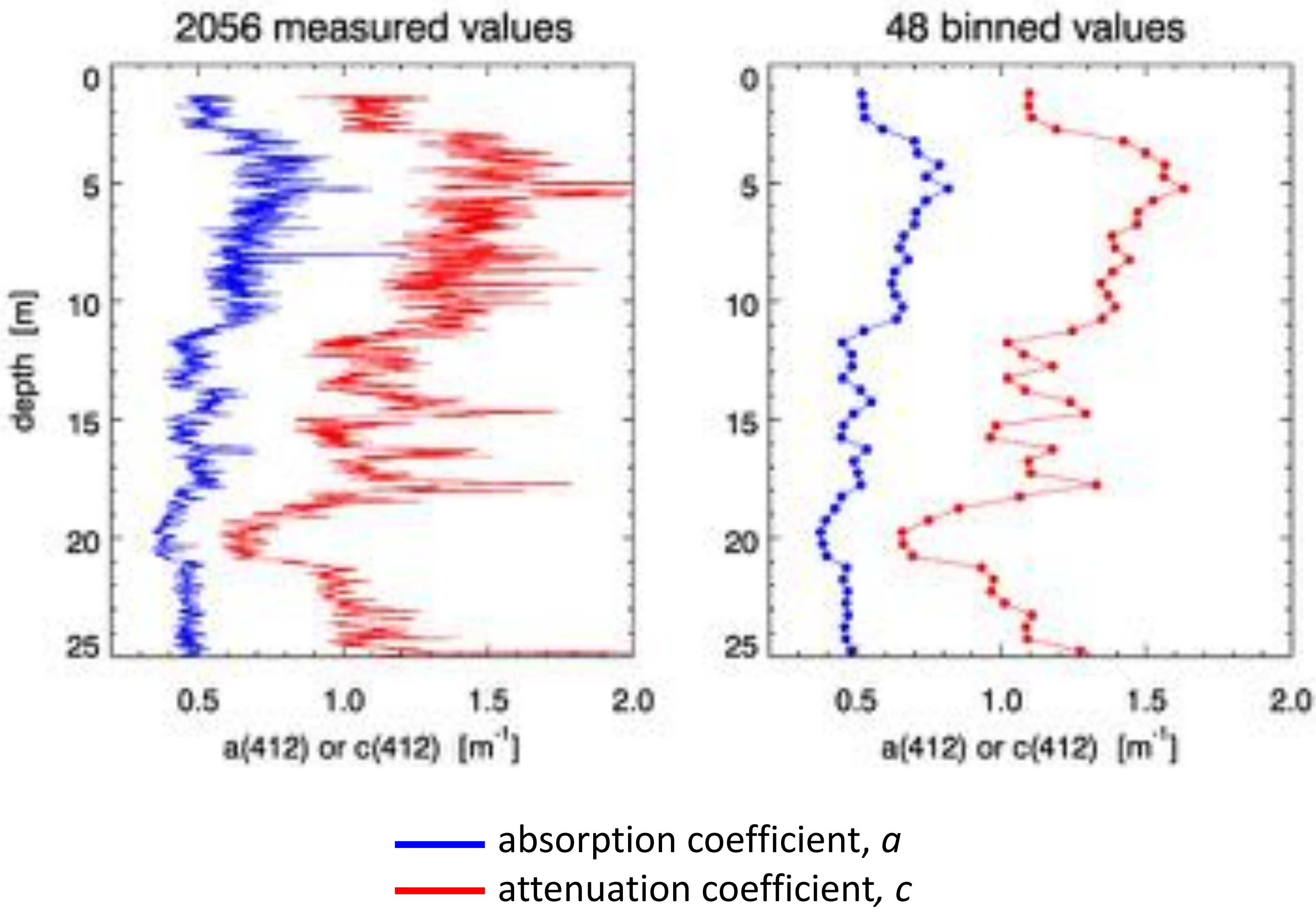
…the HydroLight model per se is a radiative transfer model, not a model of oceanic optical properties. You, the user, must supply the inherent optical properties and boundary conditions to the HydroLight core code.

HydroLight does not know the inherent optical properties, or the chlorophyll profile, or the depth, or anything else about the water body you are interested in. *You* must provide this information to HydroLight. The various IOP models, phase functions, chlorophyll data sets, ac-9 data sets, etc. that come with HydroLight are *examples* of how to provide IOP and other information to HydroLight. *You* will need to replace these example routines and data sets with your own, in order to simulate the water body of interest to you.

Garbage in = garbage out!

Measured IOP Input Data

Clean up your data before giving it to HydroLight!



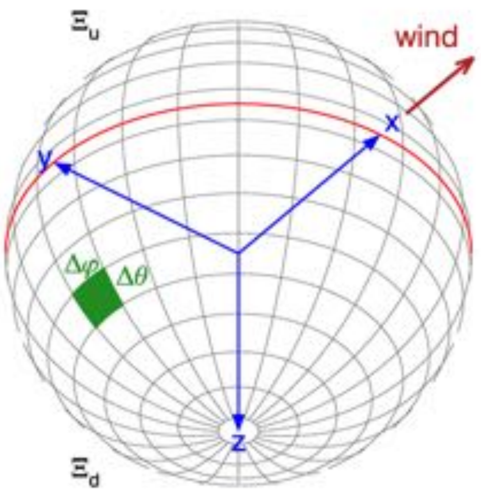
EcoLight

EcoLight is the same solution method as HydroLight but computes azimuthally averaged radiances within solid angle bands.

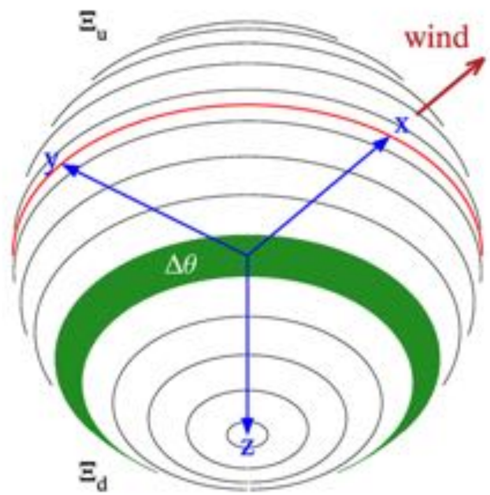
The irradiances and polar cap radiances are the same for HydroLight and EcoLight.

Diffuse attenuations (*K* values), reflectances *R*, R_{rs}, etc. are also the same.

HL



EL



$$L(z,\theta,\phi,\lambda) \rightarrow L(k,u,j) = \frac{1}{\Delta\lambda_j \, 2\pi \, \Delta\Omega_u} \int_{\Delta\lambda_j} \int_0^{2\pi} \int_{\Delta\Omega_u} L(z_k,\theta,\phi,\lambda) \sin\theta \, d\theta \, d\phi \, d\lambda$$

- EcoLight is typically 20 to 1000 times faster than HydroLight.
- To run HydroLight or EcoLight is an option at the end of the model setup.