

Comparisons & Metrological Compatibility

IOCCG Training

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Vocabulary (1)

- ❖ **measurand:** well-defined physical quantity that is to be measured
- ❖ **uncertainty of a measurement:** a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand
- ❖ **error:** difference between the measurement and the true value of the measurand (or a reference quantity value, assumed to have negligible uncertainty)
- ❖ **compatibility:**
property of a set of measurement results, such that the absolute value of the difference of any pair of measured quantity values from two different measurement results is smaller than some chosen multiple of the standard measurement uncertainty of that difference
i.e., **agreement of 2 data sets within their stated uncertainties**

Vocabulary(2)

Uncertainty associated between X_A and X_B :

$$u^2(X_B - X_A) = u^2(X_A) + u^2(X_B) - 2u(X_A)u(X_B)r(e(X_A), e(X_B))$$

Compatibility: the difference of any pair of **measured quantity values** from two different measurement results is smaller than some chosen multiple k of the **standard measurement uncertainty** of that difference

VIM 2012

$$|X_B - X_A| < k \sqrt{u^2(X_A) + u^2(X_B) - 2u(X_A)u(X_B)r(e(X_A), e(X_B))}$$

should be true for 68% of cases ($k=1$) with a normal hypothesis

Comparisons of Field Data

Example with R_{RS} data

Comparison of Field Data: Example at AAOT (1)

AAOT: Acqua Oceanographic Tower

Operations of 2 SeaPRISM systems over 2017-2023

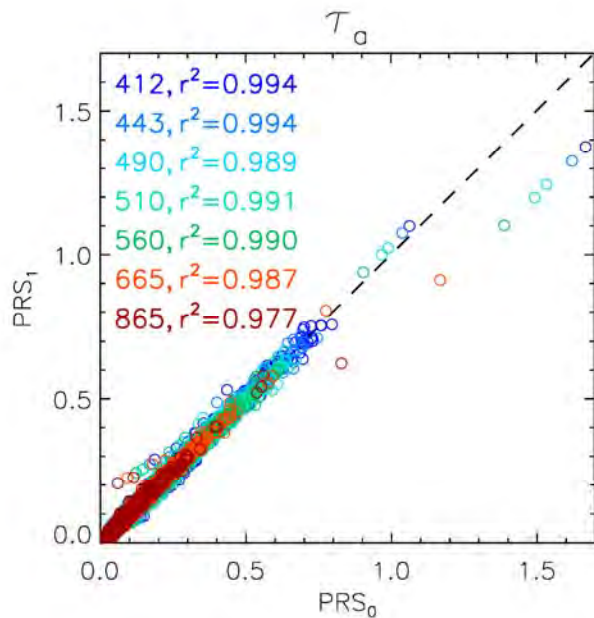


4449 pairs of coincident ($\Delta t < 10'$) measurements over 659 days of data acquisitions

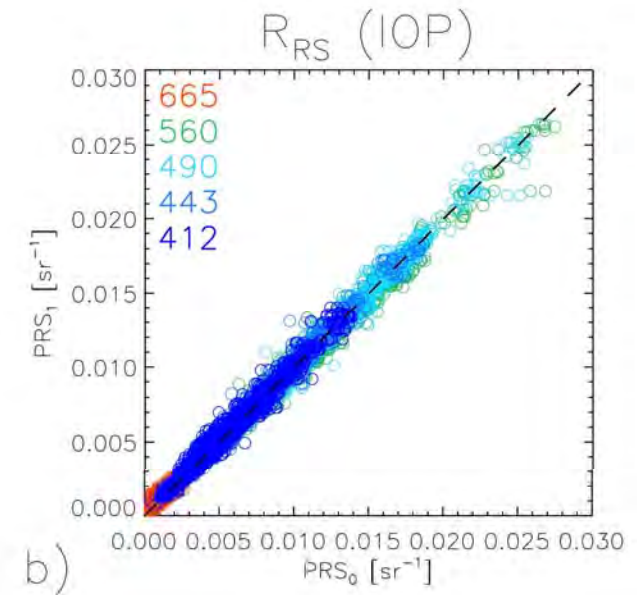
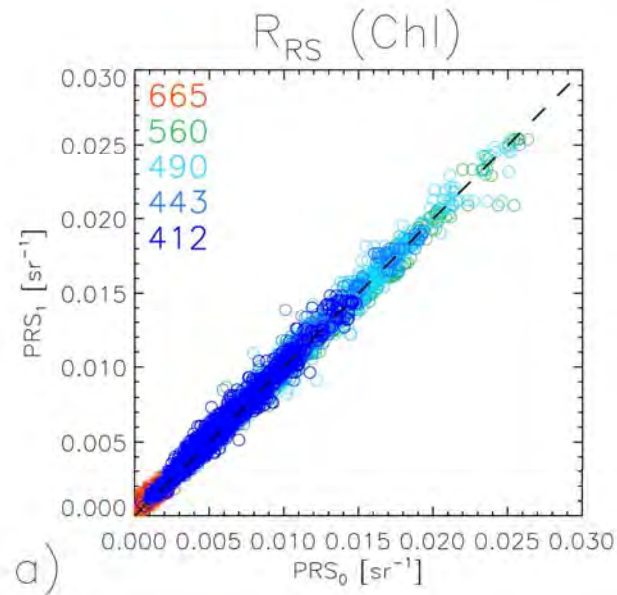


Comparison of Field Data: Example at AAOT (2)

aerosol optical thickness



remote sensing reflectance



Mélin et al. *FRS* 2024

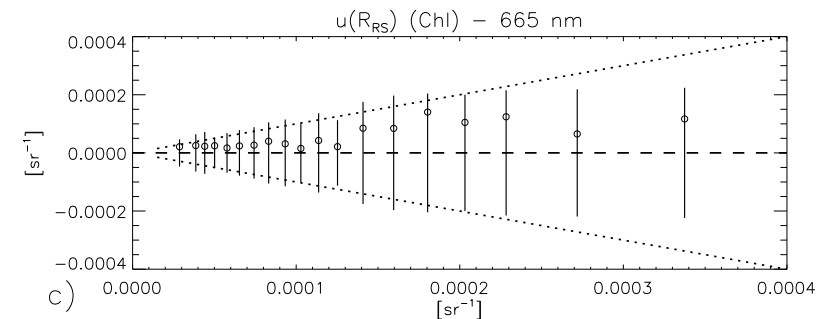
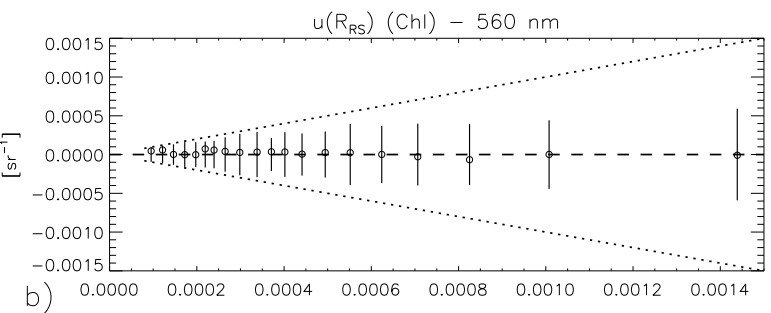
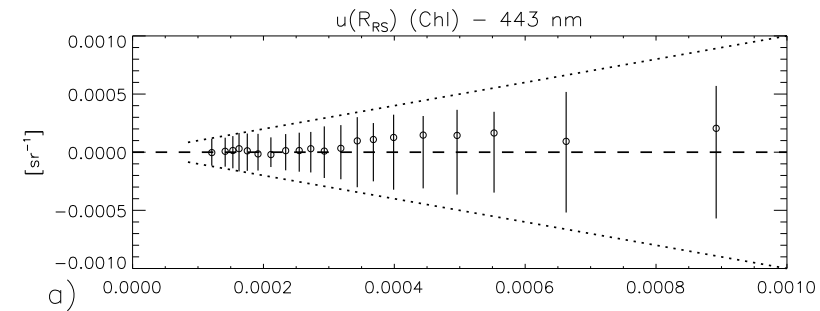
Comparison of Field Data: Example at AAOT (3)

$$u^2(X_2 - X_1) = u^2(X_1) + u^2(X_2) - 2u(X_1)u(X_2)r(e(X_1), e(X_2))$$

$$|R_{RS,1} - R_{RS,0}| < k \sqrt{u^2(R_{RS,0}) + u^2(R_{RS,1}) - 2r_\epsilon u(R_{RS,0})u(R_{RS,1})}$$

% ($k=1$)	412	443	490	560	665
$R_{RS}^{Chl}, r_\epsilon=0$	89	92	94	93	81
$R_{RS}^{Chl}, r_\epsilon=0.5$	78	82	86	83	70
$R_{RS}^{IOP}, r_\epsilon=0$	87	90	93	81	67
$R_{RS}^{IOP}, r_\epsilon=0.5$	74	80	85	65	55

Data from the two systems appear generally consistent with their stated uncertainties, indicating that they are metrologically compatible



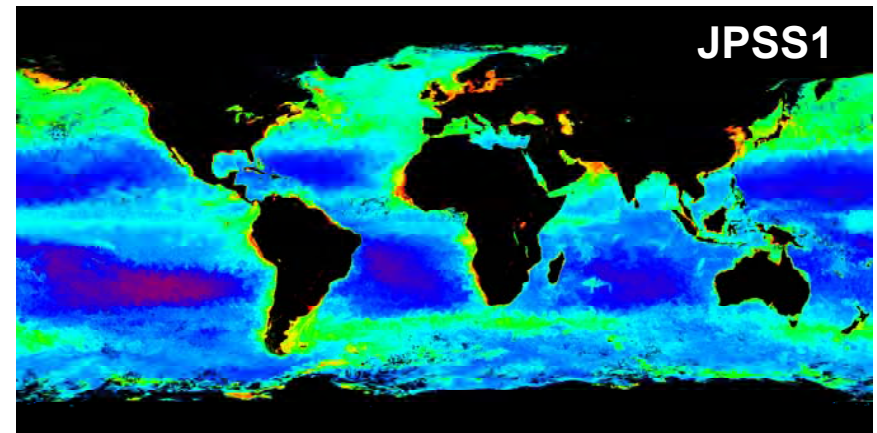
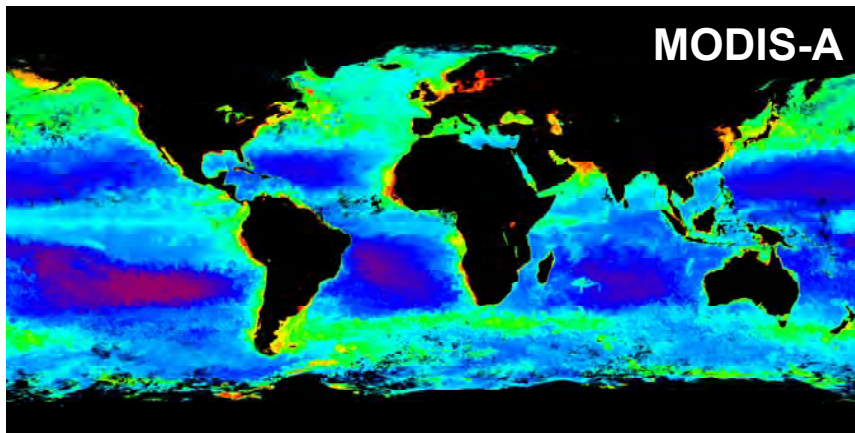
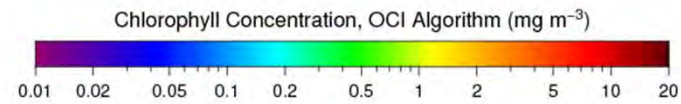
Mélin et al. *FRS* 2024



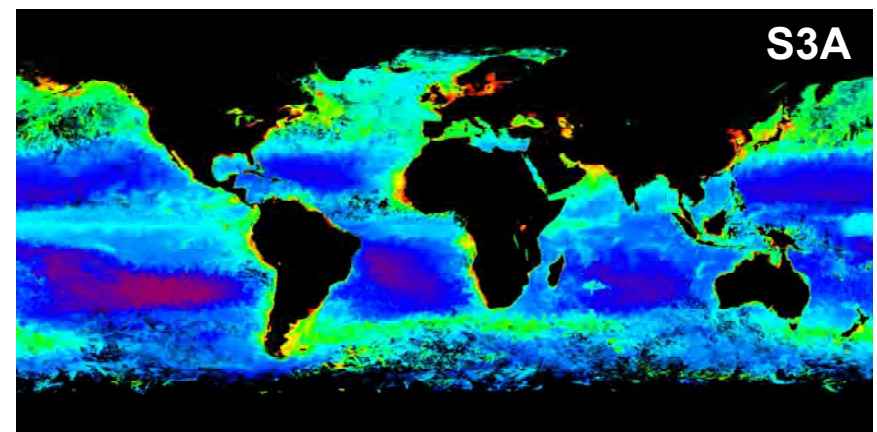
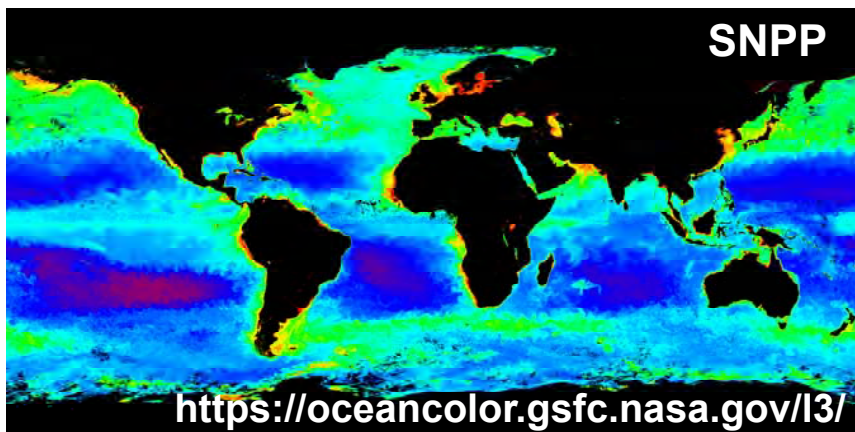
Comparisons of EO data

What we know

General consistency between missions (1)



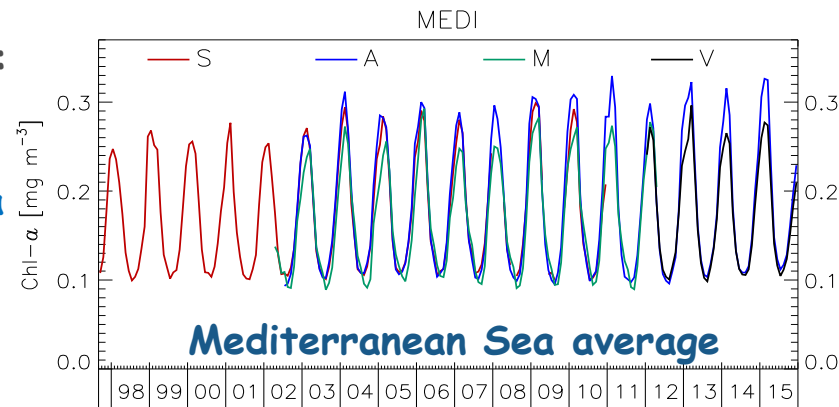
Monthly
Chl-a
Mar. 2018



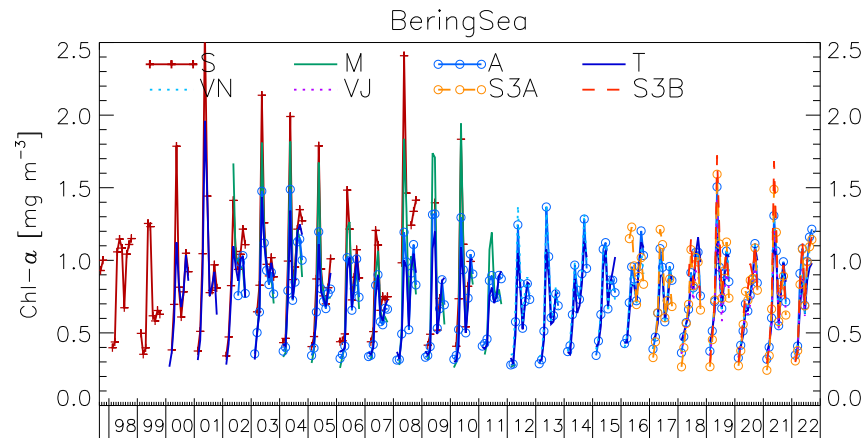
General consistency between missions (2)

NASA products:

- **SeaWiFS**
- **MERIS**
- **MODIS-Aqua**
- **VIIRS**



- S: SeaWiFS
- M: MERIS
- A: MODIA-Aqua
- T: MODIS-Terra
- VN: VIIRS-SNPP
- VJ: VIIRS-JPSS1
- S3A: OLCI S-3A
- S3B: OLCI S-3B



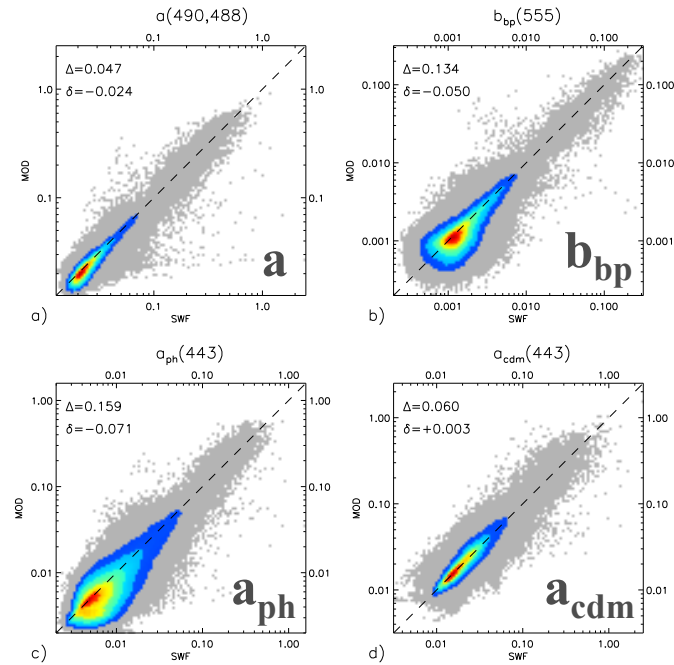
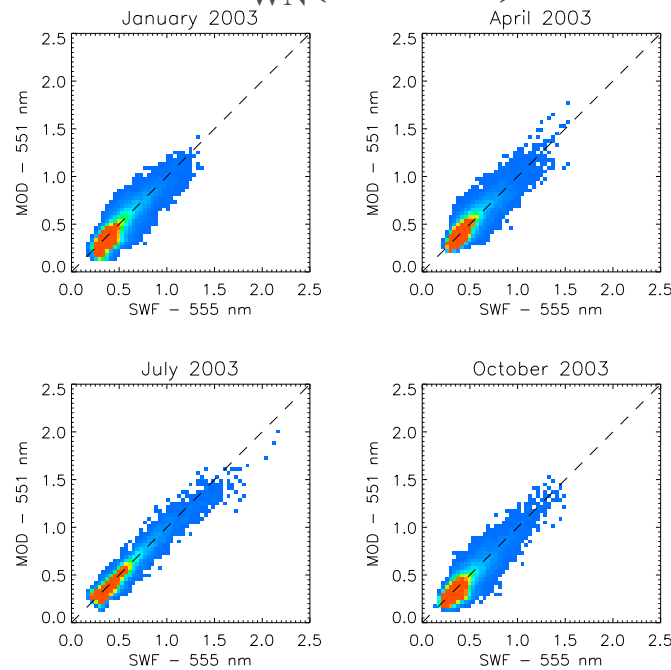
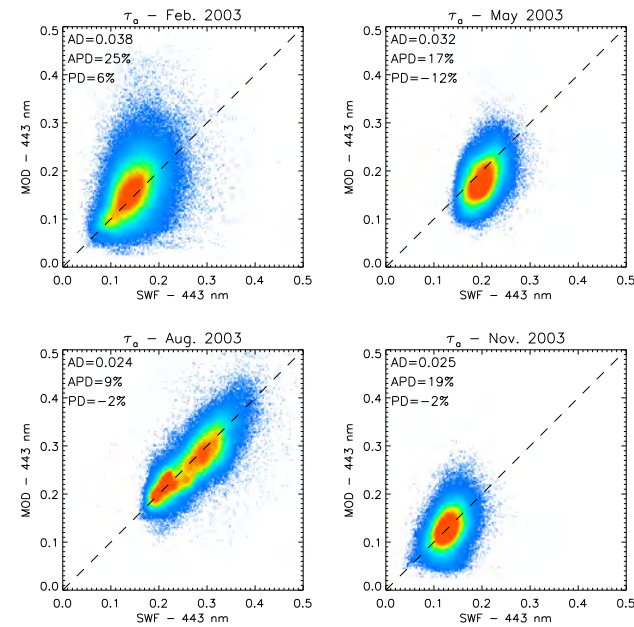
General consistency between missions (3)

MODIS-A vs SeaWiFS

$\tau_a(443)$

$L_{WN}(551/555)$

IOPs



Mediterranean Sea
Mélin et al., RSE 2007

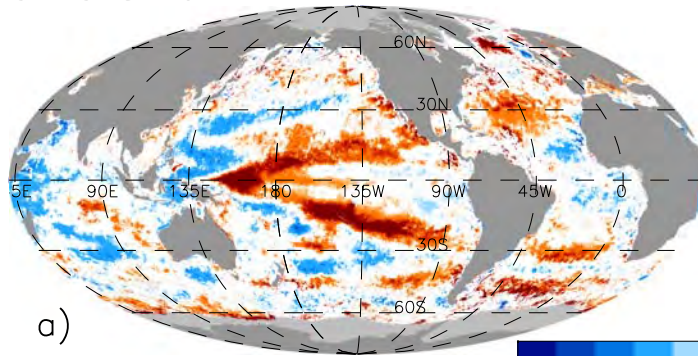
Mediterranean Sea
Mélin et al., ASR 2009

Adriatic Sea
Mélin et al., OS 2011

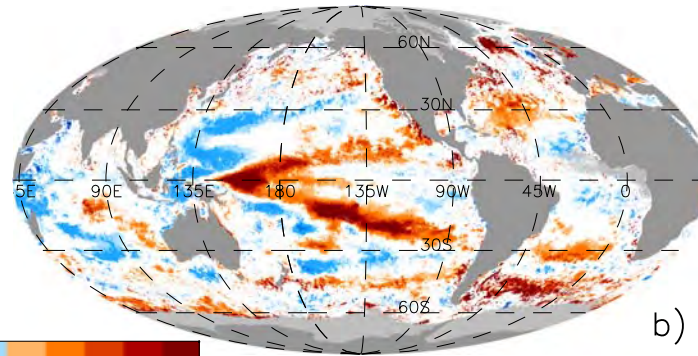
General consistency between missions (4)

Chl-a 10-year trend
Aug. 2002
Jul. 2011

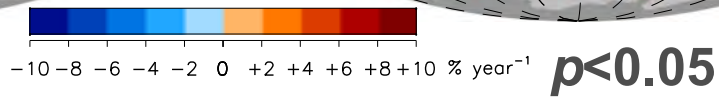
MODIS-A



Mélin et al., *RSE* 2017

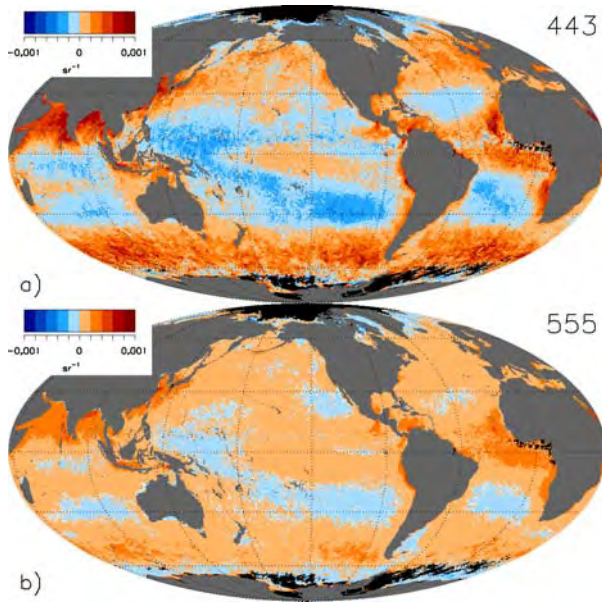


MERIS



Ocean Color products (R_{RS} , IOPs, Chl-a) from various missions show differences varying:

in space

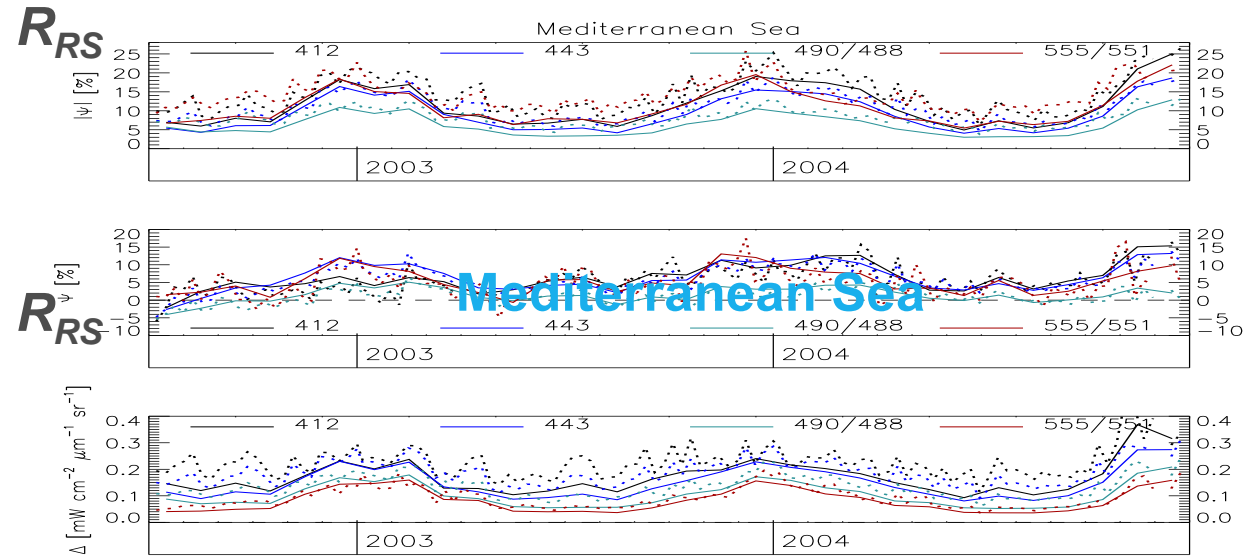


SeaWiFS-MODIS

$$\delta = \sum_{i=1}^N (y_i - x_i)$$

Mélin et al., RSE 2016

in time



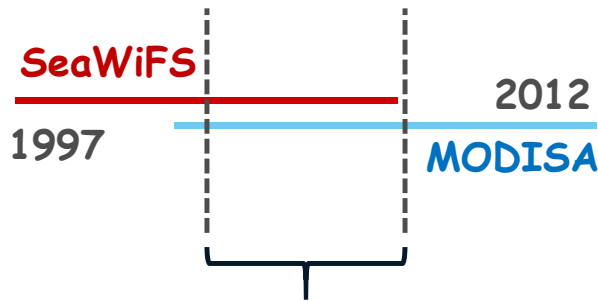
$$|\psi| = \sum_{i=1}^N \frac{2|y_i - x_i|}{x_i + y_i} \quad \psi = \sum_{i=1}^N \frac{2(y_i - x_i)}{x_i + y_i}$$

Mélin et al., ASR 2009

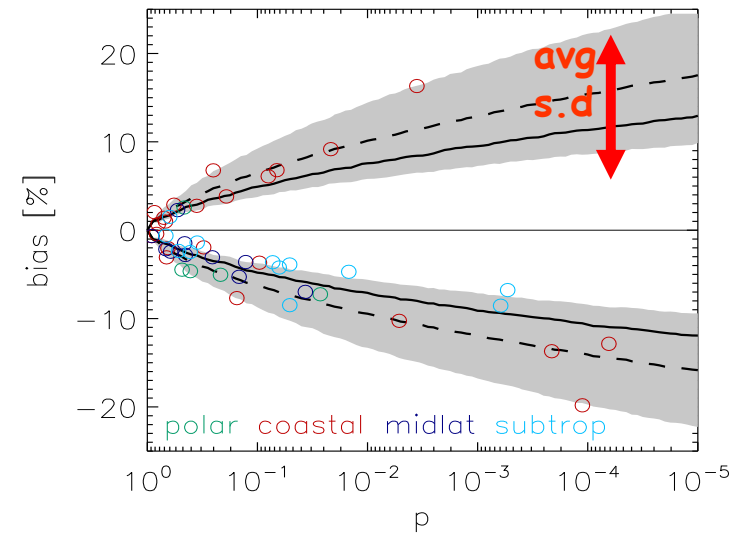
Small systematic inter-mission differences are sufficient to introduce spurious trends

Comparing **trend slopes** of merged products affected by a bias wrt those obtained for a series of reference

p: level of significance (*t*-test) quantifying the degree to which 2 trends differ



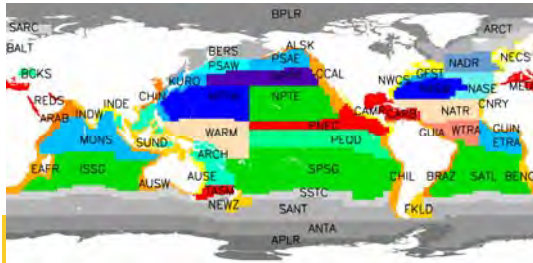
Mélin, *IJRS* 2016



50% 90% 99% p

$$x_{MRG}(m) = \frac{1}{2} [x_S(m) + (1 + \%bias/100) x_{A,corr}(m)]$$

%bias: [-50% to +50%]



differences of 5-6%
 → significant differences in slopes ($p < 0.05$)

Framework to characterize differences between products (e.g., R_{RS})

averaging N_A pixel-values

$$\langle R_{RS}^A(\lambda_A) \rangle = \frac{1}{N_A} (R_{RS,1}^A(\lambda_A) + \dots + R_{RS,ip}^A(\lambda_A) + \dots + R_{RS,N_A}^A(\lambda_A)) + q_0$$

model error

- binning

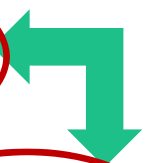
$$\frac{\partial \delta R_{RS}}{\partial R_{RS}^A} = -1$$

$$\frac{\partial \langle R_{RS}^A \rangle}{\partial R_{RS,ip}^A} = \frac{1}{N_A}$$

$$u(R_{RS,ip}^A)$$

error on R_{RS} from mission A

error correlation



error on R_{RS} from mission B

$$\delta R_{RS}(\lambda_A) = \langle R_{RS}^B(\lambda_A) \rangle - \langle R_{RS}^A(\lambda_A) \rangle + q_0$$

model error

- geolocation
- mismatch
- time difference

$$\frac{\partial R_{RS}^B(\lambda_A)}{\partial R_{RS}^B(\lambda_b)}$$

$$u(R_{RS,ip}^B)$$

$$\frac{\partial \delta R_{RS}}{\partial R_{RS}^B} = 1$$

$$\frac{\partial \langle R_{RS}^B \rangle}{\partial R_{RS,ip}^B} = \frac{1}{N_B}$$

$$R_{RS,ip}^B(\lambda_A) = f(R_{RS,ip}^B(\lambda_B)) + q_0$$

$$u(q_0)$$

$$\langle R_{RS}^B(\lambda_A) \rangle = \frac{1}{N_B} (R_{RS,1}^B(\lambda_A) + \dots + R_{RS,ip}^B(\lambda_A) + \dots + R_{RS,N_B}^B(\lambda_A)) + q_0$$

model error

- binning

model error

- band-shifting
- spectral response

averaging N_B pixel-values

The Case of Composites

$$\delta R_{RS}(\lambda_A) = \langle R_{RS}^B(\lambda_A) \rangle - \langle R_{RS}^A(\lambda_A) \rangle + q_0$$

$u(q_0)$ — model error

- geolocation mismatch
- time difference

- Different times of the day for a daily datum (polar-orbiting)
- Grid points incompletely/variably filled
- Different days for a time composite

Comparisons of EO data

Examples with R_{RS} at validation sites (AERONET-OC)

Source of field R_{RS} data: AERONET-OC

GDLT (2005) HLT (2006) IRLT (2018)



GLR/S7 (2011)



- JRC Marine Sites
- Active Marine Sites
- Active Lake Sites
- Decommissioned Sites

PLOCAN (2022)



CSP (2019)



AAOT (2002)

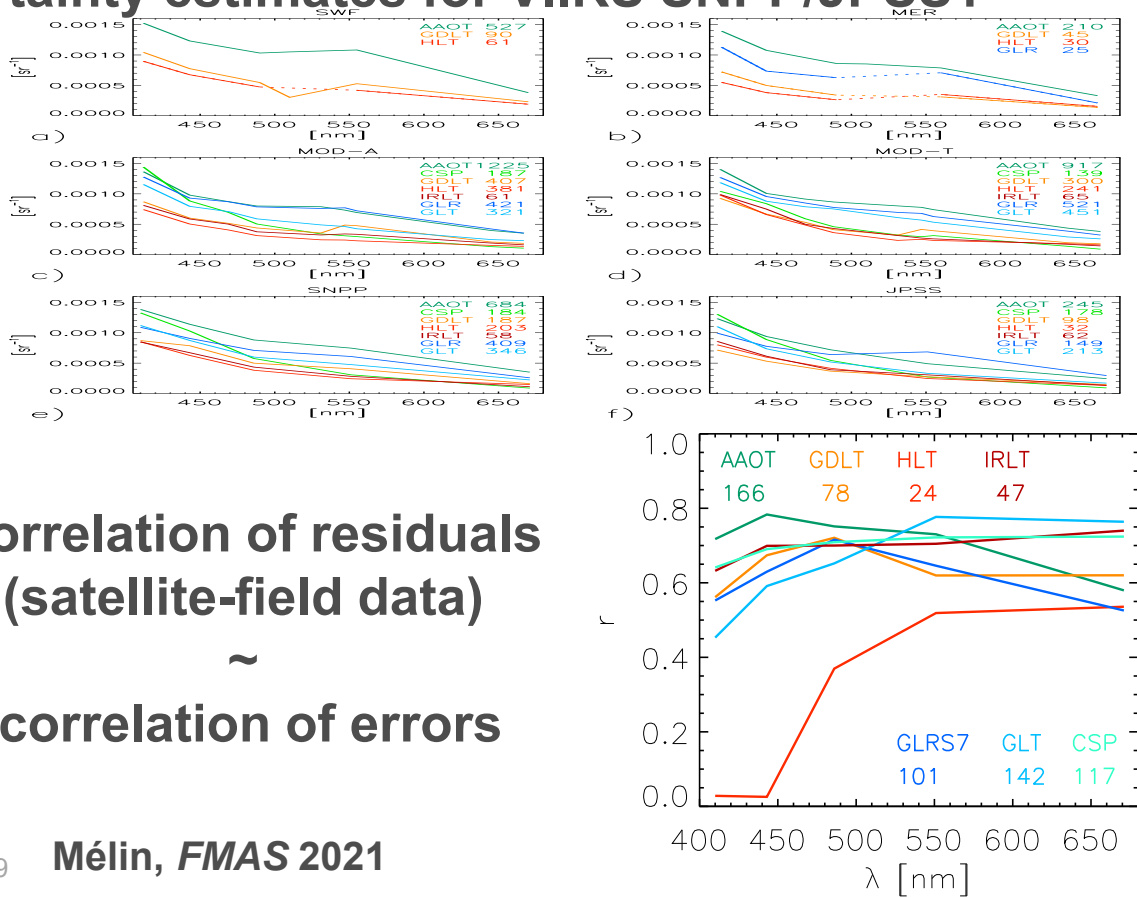


GLT (2014)

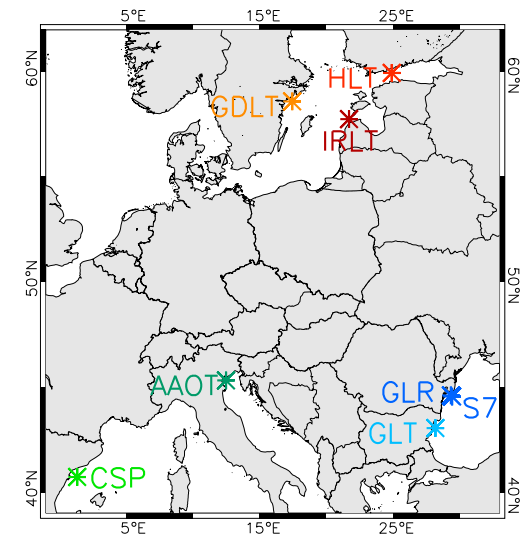


Comparison of EO Data: Example with VIIRS (1)

Uncertainty estimates for VIIRS SNPP/JPSS1

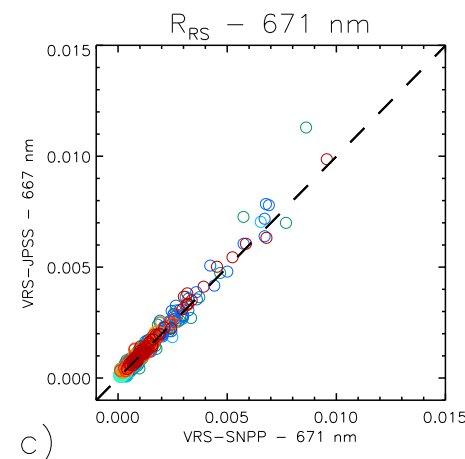
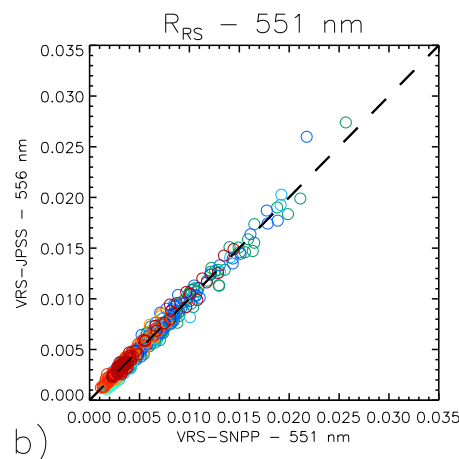
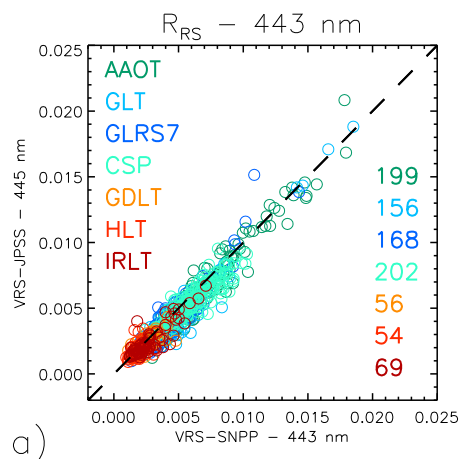


correlation of residuals
(satellite-field data)
~
correlation of errors



Comparison of EO Data: Example with VIIRS (2)

SNPP
VS
JPSS1



Fraction of data verifying ($k=1$) (no representation error):

$$|R_{RS}^B - R_{RS}^A| < k \sqrt{u^2(R_{RS}^A) + u^2(R_{RS}^B) - 2u(R_{RS}^A)u(R_{RS}^B)r(e(R_{RS}^A), e(R_{RS}^B))}$$

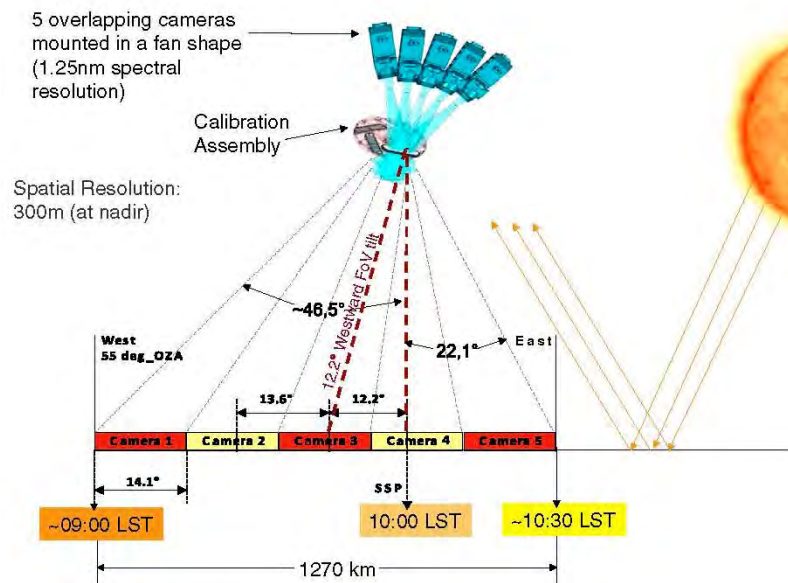
AAOT	410	443	486	551	671
r=0 (%)	93	90	93	92	95
r≠0 (%)	69	60	70	76	89

GLT	410	443	486	551	671
r=0 (%)	91	87	87	90	95
r≠0 (%)	85	67	71	67	81

Comparison of EO Data: Example with OLCI (1)

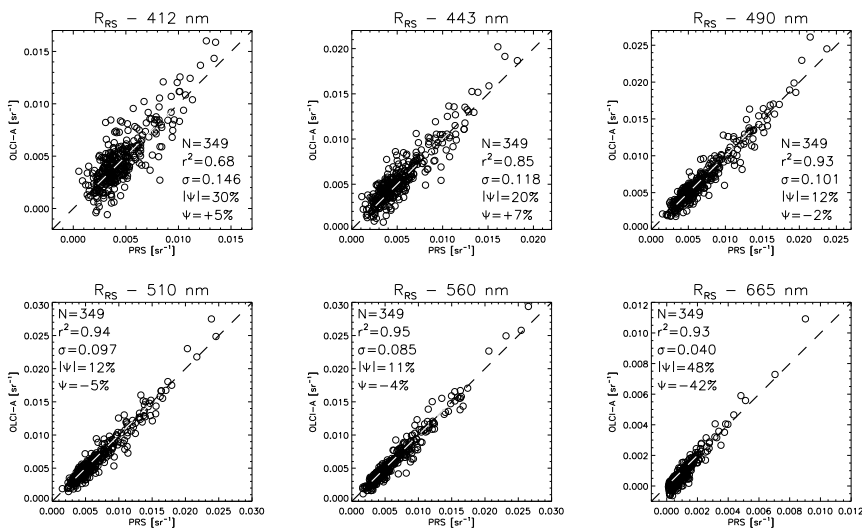
Cam.	1	2	3	4	5	
S3A	24%	26%	15%	21%	13%	N=349
S3B	30%	20%	16%	20%	13%	N=220

(out of tandem)

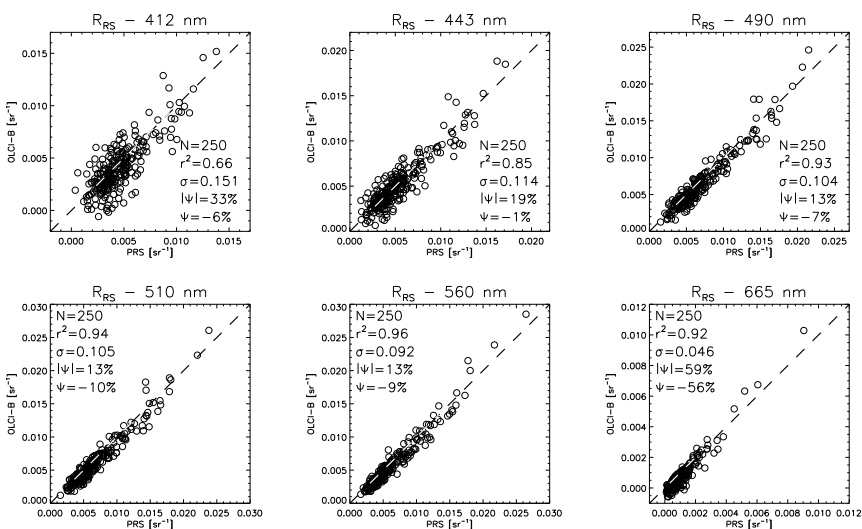


Donlon et al. RSE (2012)

S3A

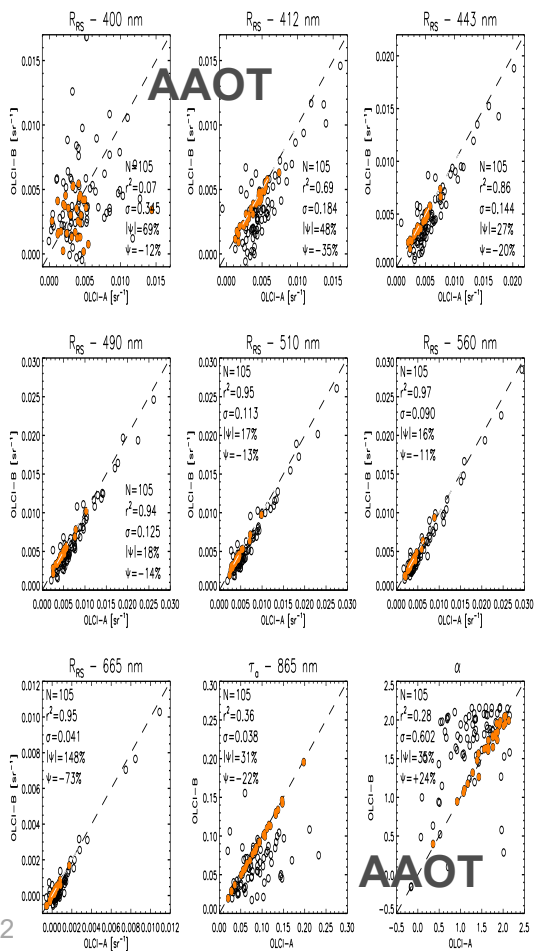


S3B



Comparison of EO Data: Example with OLCI (2)

OLCI S3A vs S3B



		S3A						
S3B	Cam.	1	2	3	4	5	=75%	
	1				5.5%	46.6%		23.2%
	2			1.4%				11.0%
	3							
	4					2.7%		
	5	8.2%						1.4%



Beware of conditions represented by the comparison data set before reaching conclusions

Orange:
tandem phase

Collocation Statistics

Collocation (1): with Reference Data

For $i=1,N$ validation points:

$$\begin{cases} x_i = r_i + \gamma_i \\ y_i = \beta r_i + \delta + \varepsilon_i \end{cases}$$

r : reference values (“truth”)

x : in-situ measurements **with σ_γ known**

γ : associated random error term (mean=0, un-correlated)

y : satellite values, **with σ_ε unknown**

ε : associated random error term (mean=0, un-correlated)

δ : additive bias

β : multiplicative bias

➔
$$\sigma_\varepsilon^2 = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2 - \sigma_\gamma^2} \quad (\delta \text{ and } \beta \text{ can also be computed})$$

Collocation (2): with 2 Similar Data Sets

For $i=1,N$ comparison points:

$$\begin{cases} x_i = r_i + \gamma_i \\ y_i = \beta r_i + \delta + \varepsilon_i \end{cases}$$

$$\eta = \frac{\sigma_\varepsilon}{\sigma_\gamma}$$

r : reference values (“truth”)

x : in-situ measurements **with σ_γ unknown**

γ : associated random error term (mean=0, un-correlated)

y : satellite values, **with σ_ε unknown**

ε : associated random error term (mean=0, un-correlated)

δ : additive bias

β : multiplicative bias

$$\beta = \frac{-\eta^2 \sigma_x^2 + \sigma_y^2 + \sqrt{(\sigma_y^2 - \eta^2 \sigma_x^2)^2 + 4\sigma_{xy}^2 \eta^2}}{2\sigma_{xy}}$$

Slope of Model II linear regression
Legendre & Legendre (1998)

$$\begin{cases} \sigma_\gamma^2 = \frac{1}{2} \left[\sigma_x^2 + \frac{\sigma_y^2}{\eta^2} - \sqrt{\left(\frac{\sigma_y^2}{\eta^2} - \sigma_x^2\right)^2 + 4\sigma_{xy}^2/\eta^2} \right] \\ \sigma_\varepsilon^2 = \frac{1}{2} \left[\eta^2 \sigma_x^2 + \sigma_y^2 - \sqrt{(\sigma_y^2 - \eta^2 \sigma_x^2)^2 + 4\sigma_{xy}^2 \eta^2} \right] \end{cases}$$

Collocation (3): with 2 Similar Data Sets

By the way:

$$\Delta_c^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}$$

$$\Delta_c^2 = (\beta - 1)^2 \sigma_x^2 + \beta(2 - \beta) \sigma_y^2 + \sigma_\varepsilon^2 \longrightarrow \sigma_y^2 + \sigma_\varepsilon^2$$

$$\text{if } \beta \longrightarrow 1$$

Comparison is the 3rd principle of metrology

Shift to variable-centric view (consistent multi-mission data records) is still a challenge

Comparison of data has enormous potential (partly untapped)

Bonus on Collocation Statistics

Collocation : with 2 Similar Data Sets, with error correlation

For $i=1,N$ comparison points:

$$\begin{cases} x_i = r_i + \gamma_i \\ y_i = \beta r_i + \delta + \varepsilon_i \end{cases}$$

$$\eta = \frac{\sigma_\varepsilon}{\sigma_\gamma}$$

r : reference values (“truth”)

x : in-situ measurements **with σ_γ unknown**

γ : associated random error term (mean=0, **correlated**)

y : satellite values, **with σ_ε unknown**

ε : associated random error term (mean=0, **correlated**)

δ : additive bias

β : multiplicative bias

$$\beta = \frac{\sigma_y^2 - \eta^2 \sigma_x^2 + \sqrt{(\sigma_y^2 - \eta^2 \sigma_x^2)^2 + 4(\sigma_{xy} - r_{\varepsilon\gamma} \eta \sigma_x^2)(\eta^2 \sigma_{xy} - r_{\varepsilon\gamma} \eta \sigma_y^2)}}{2(\sigma_{xy} - r_{\varepsilon\gamma} \eta \sigma_x^2)}$$

$r_{\varepsilon\gamma}$: Correlation between ε_i and γ_i

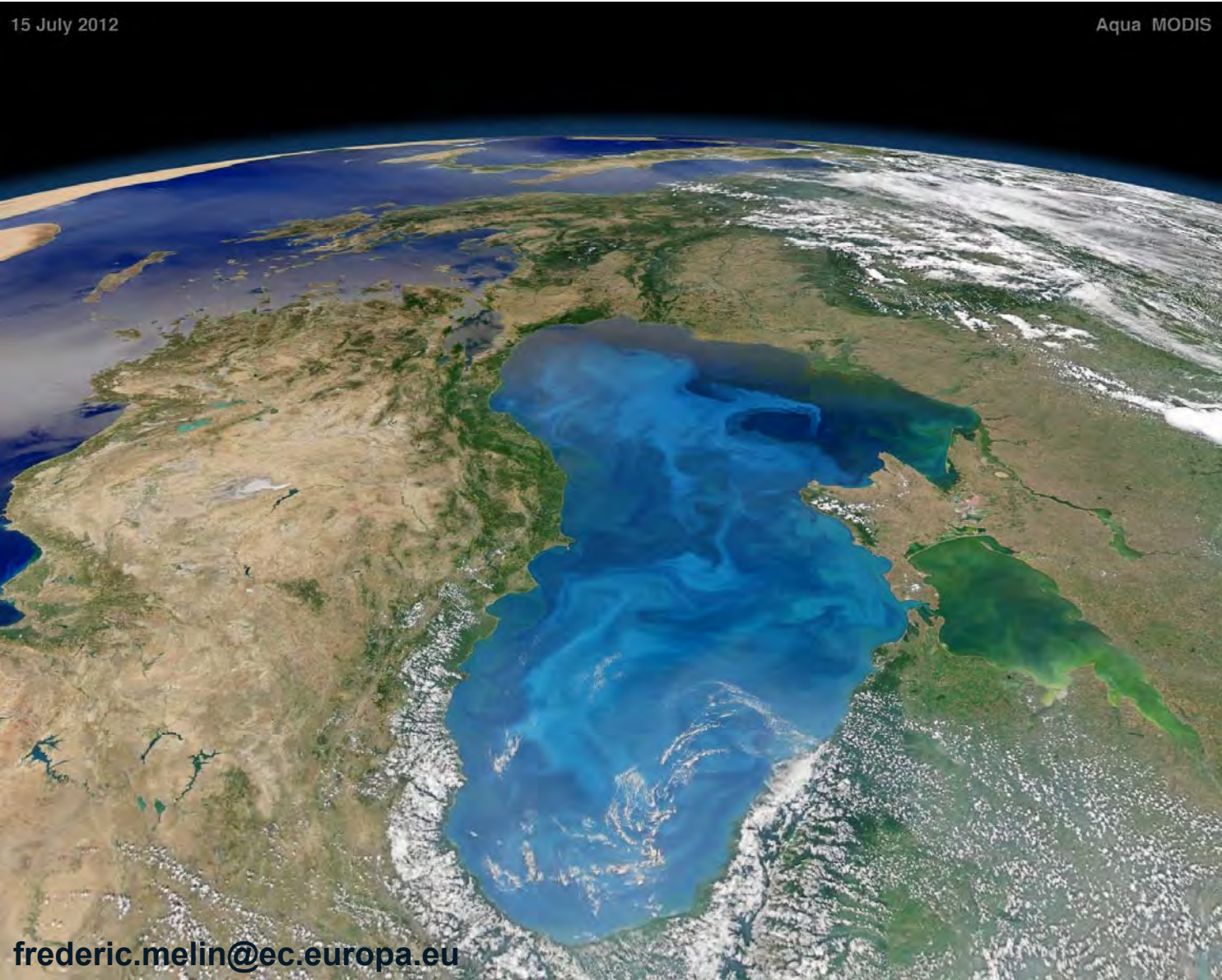
Collocation : with 2 Similar Data Sets, with error correlation

$$\left\{ \begin{array}{l} \sigma_{\gamma}^2 = \frac{\beta\sigma_x^2 - \sigma_{xy}}{\beta - \eta r_{\varepsilon\gamma}} \\ \sigma_{\varepsilon}^2 = \frac{\sigma_y^2 - \beta\sigma_{xy}}{1 - \beta r_{\varepsilon\gamma}/\eta} \end{array} \right.$$

$$\Delta_c^2 = (\beta - 1)^2 + [\beta(2 - \beta) + \eta^2 - 2\eta r_{\varepsilon\gamma}] \sigma_{\gamma}^2$$

$$\longrightarrow \sigma_{\gamma}^2(1 + \eta^2 - 2\eta r_{\varepsilon\gamma}) \quad \text{if } \beta \longrightarrow 1$$

$$\longrightarrow \sigma_{\gamma}^2(2 - 2r_{\varepsilon\gamma}) \quad \text{if } \beta \longrightarrow 1 \text{ and } \eta = 1$$



Thank you!