Introduction to Metrology
The science of measurements

IOCCG Training

22-23 Apr. 2024

Frédéric Mélin

European Commission

Metrology & Ancient Egypt

Royal cubit, tomb of Maya, treasurer of Tutankhamun, 14th C. BC (Louvre)
Monnier et al. 2016

Pharaoh / Royal Cubit ↔ **Royal Architect** ↔ **Foreman of each construction site** ↔ **Workers**

- **Dissemination of a unit**
- **Measurement standard (Royal Cubit)**
- **Traceability of length measurements to a reference**
- **Calibration of measuring systems (comparison of cubit sticks to the reference at each full Moon)**
- **Standardization of measurement procedures over all Egypt**

De Bièvre AQA 2005

Metrology: in 4 dates (1)

- **1799: Definition of the "Mètre définitif" and adoption of the metric system in France**

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Metrology: in 4 dates (2)

- **1875: Metre convention ("convention du mètre")**
  creation of BIPM (International Bureau of Weights & Measures)

Bureau International des Poids et Mesures

Metrology: in 4 dates (3)

- **1960: International System of Units (SI) established by the General Conference on Weights & Measures**

- **2019: Four units re-defined in terms of universal constants (kg, A, K, mol)**

European Commission

National Metrology Institutes (NMIs)

- **Argentina: Instituto Nacional de Tecnología Industrial (INTI)**

https://www.argentina.gob.ar/inti

- **Brazil: Instituto Nacional de Metrologia, Qualidade e Tecnologia (INMETRO)**

https://www.gov.br/inmetro/pt-br

Metrology: 3 principles

- **Traceability:**
  - unit definition (SI)
  - primary standards (BIPM, NMIs)
  - secondary standards
  - calibration (users)
  - measurements

increasing uncertainties

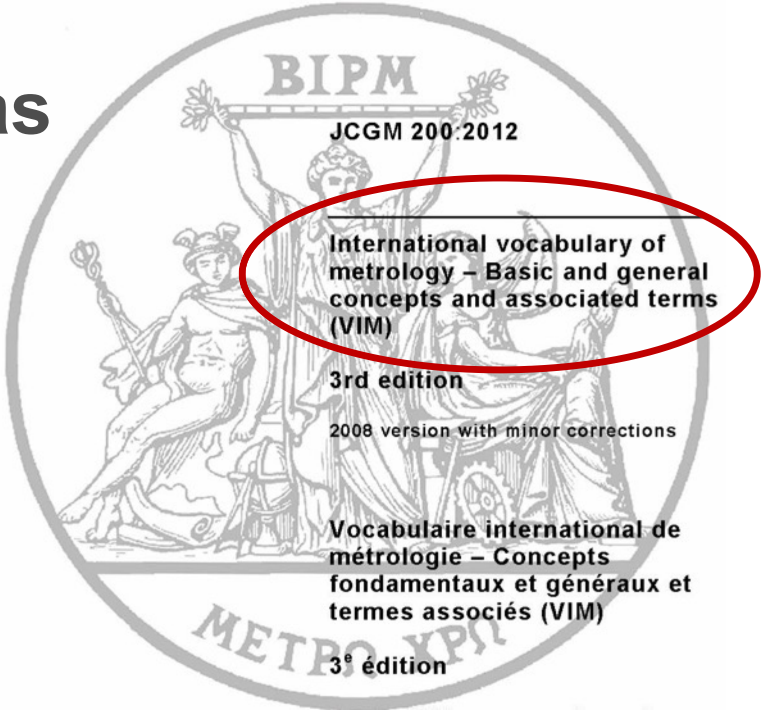
- **Uncertainty analysis**

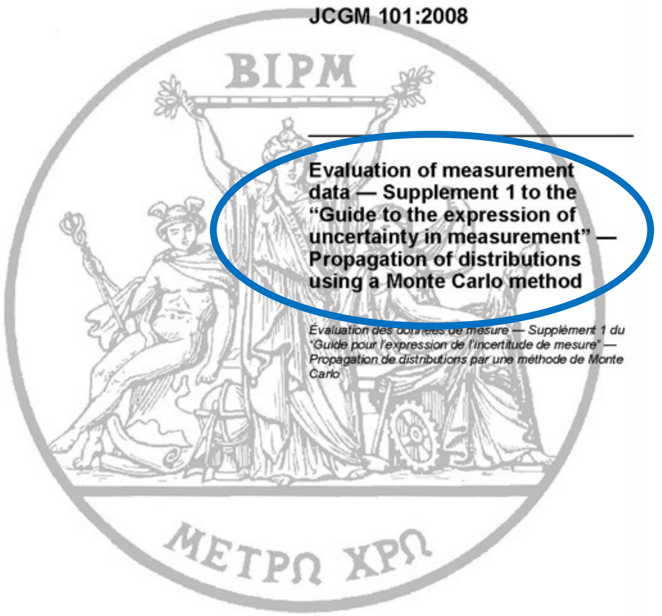
https://www.bipm.org/en/committees/jc/jcgm/publications

- **Comparison**

Fast-forward to 21st Century….

- Metrology has come of age as the science of measurement
- Definition of terminology
- Development of methods
- Uncertainty as a key element to allow an informed use of data





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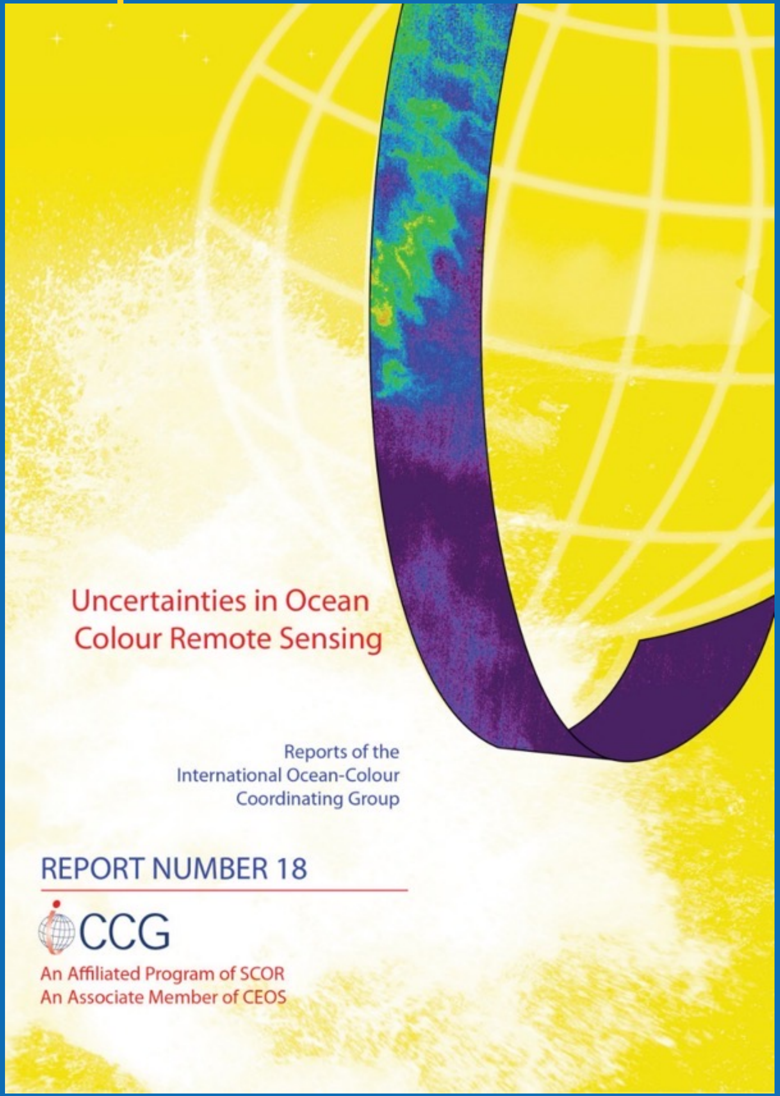
Metrology & Ocean Colour Remote Sensing



IOCCG Report #18 (2019)

- Terminology and Main Principles
- Sources of Uncertainties
- Uncertainty Estimates
- Representation and Distribution of Uncertainties
- Requirements for Different Applications
- Recommendations

https://ioccg.org/what-we-do/ioccg-publications/ioccg-reports/

Vocabulary

- **measurand**: well-defined physical quantity that is to be measured
- **uncertainty of a measurement**: a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand

dispersion => uncertainty expressed as a standard deviation (standard uncertainty ***u***)

y-U < Y < y+U with **U=*k.u*** ; ***k*: coverage factor**

evaluation of uncertainty:

- Type A: based on a statistical analysis of a series of measurements
- Type B: based on other methods (expert judgment, specifications, …)

Vocabulary

- **measurand**: well-defined physical quantity that is to be measured
- **uncertainty of a measurement**: a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand
- **traceability**: property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty
- **error:** difference between the measurement and the true value of the measurand (or a reference quantity value, assumed to have negligible uncertainty)
- **accuracy:** closeness of agreement between a measured quantity value and the true value of the measurand

ERROR

≠

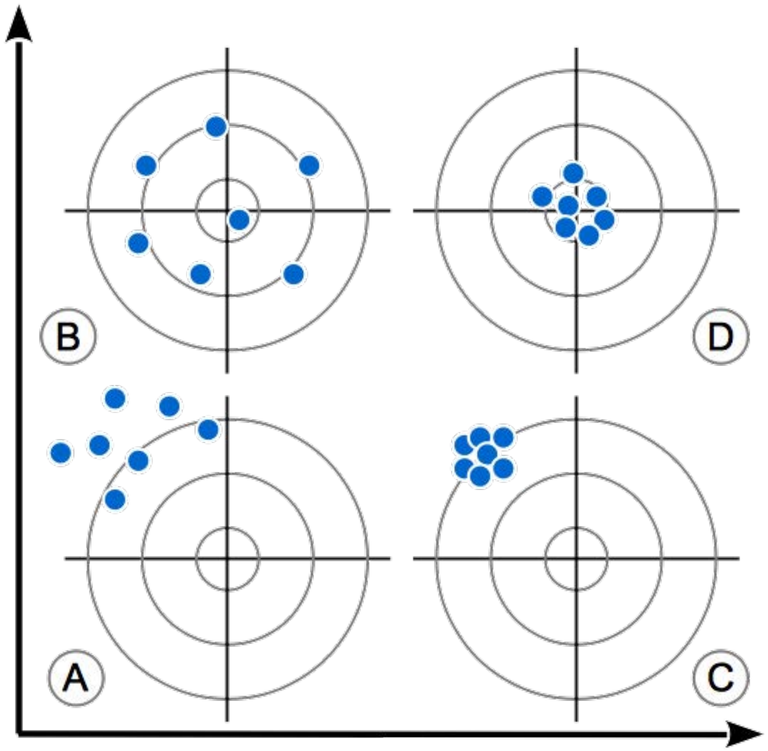
UNCERTAINTY

Vocabulary

- **precision**: closeness of agreement between values obtained by replicate measurements

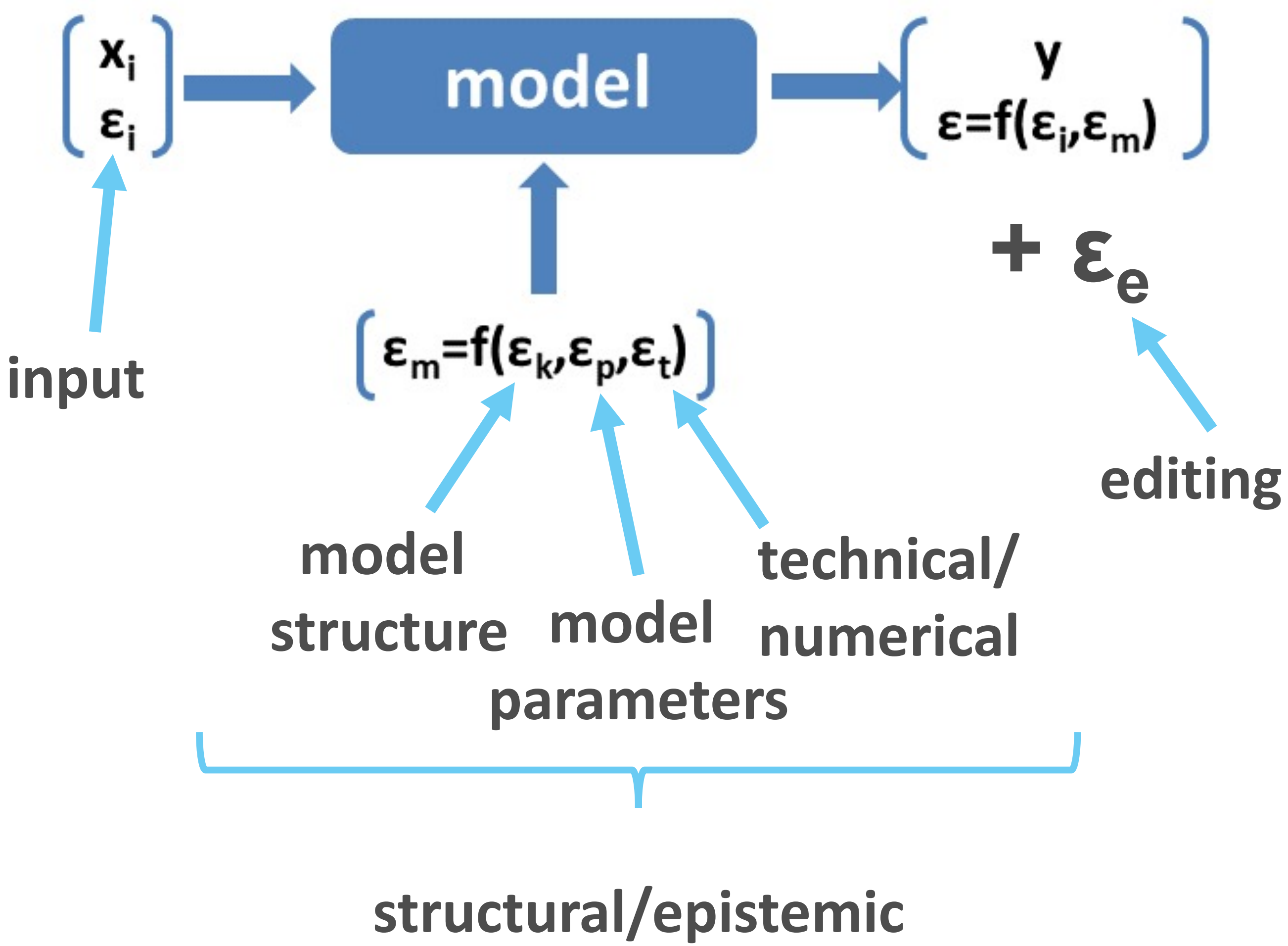
→ **repeatability, reproducibility**

- **verification**: process providing objective evidence that stated uncertainty fulfill specific requirements



- **compatibility***:* property of a set of measurement results, such that the absolute value of the difference of any pair of measured quantity values from two different measurement results is smaller than some chosen multiple of the standard measurement uncertainty of that difference (agreement within stated uncertainties)

Types of errors

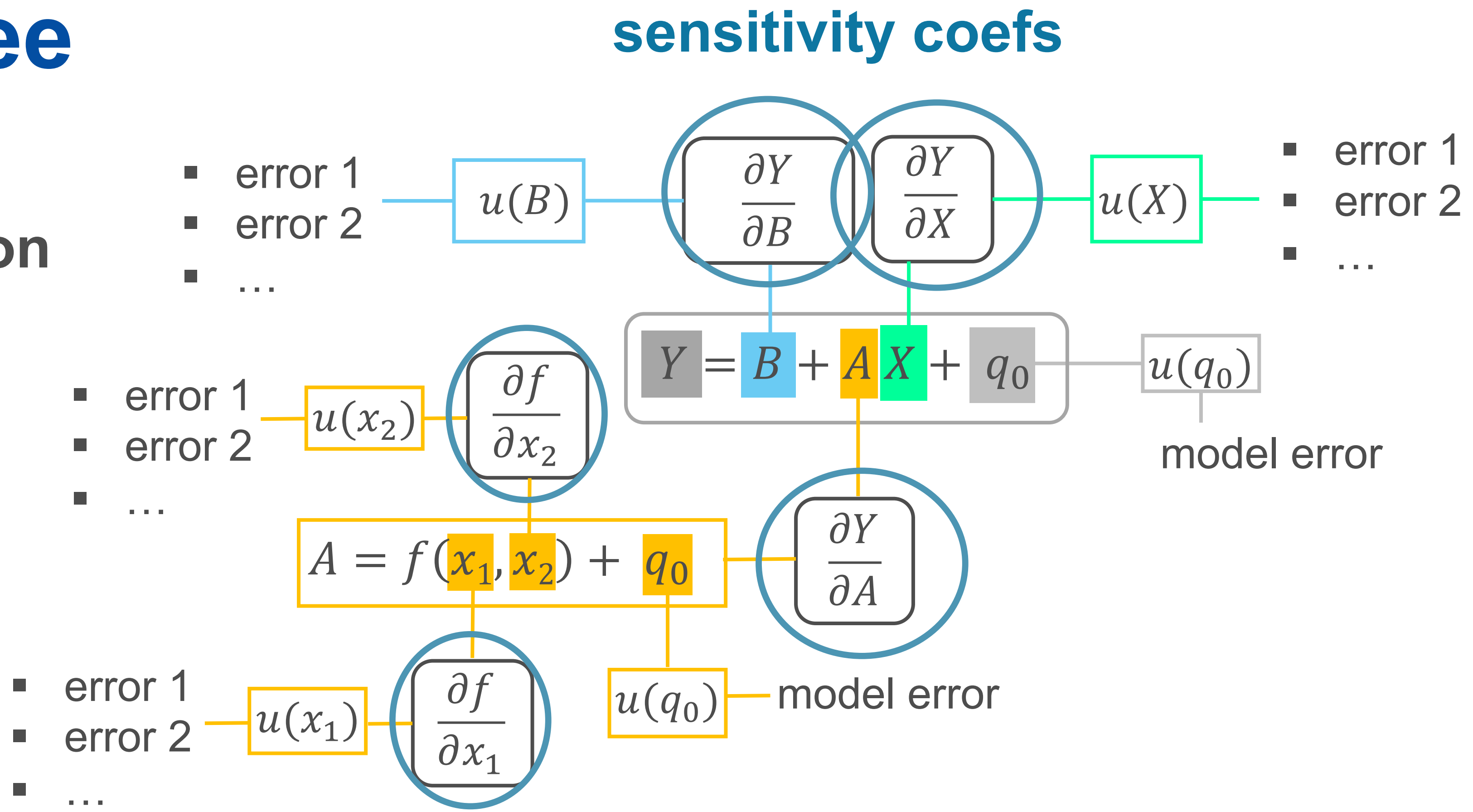


- Random
- Locally systematic
- Systematic

- Uncorrelated
- Spectrally correlated
- Spatially correlated
- …

Uncertainty tree

Measurement model/equation defined as a function of **influence quantities**



For each element:

- **Understand the process**
- **Identify assumptions made in modelling**
- **Characterize the error terms (distribution, …)**
- **Identify error correlation terms (time, space, spectrally, …)**

Law of propagation of Uncertainties

$$y = f(x_i)_{i=1,N}$$
$$y + \varepsilon = f(x_i + \varepsilon_i)_{i=1,N}$$

$$\varepsilon = \sum_{i=1}^{N} \frac{\partial f}{\partial x_i}(x_i)\varepsilon_i$$

Taylor expansion for small perturbations

combined uncertainty of measurand *y*:

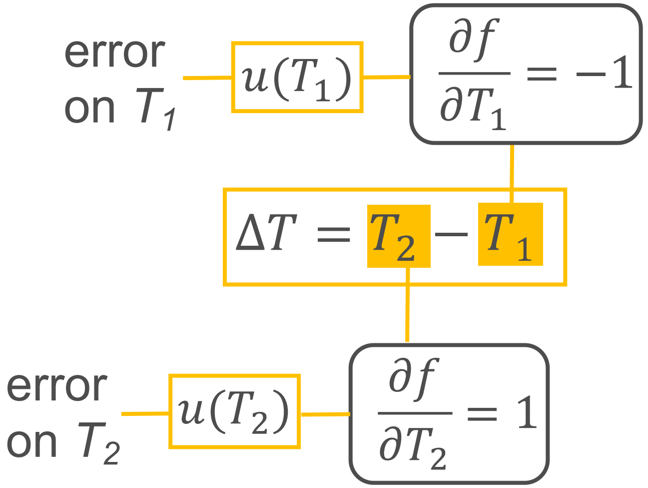
$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{x_j}u(x_i, x_j)$$

covariance terms

$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{x_j}r_{ij}u(x_i)u(x_j)$$

Impact of error correlation

Example of a temperature gradient:



$$u_c^2(\Delta T) = u^2(T_1) + u^2(T_2) - 2r_{12}u(T_1)u(T_2)$$

$$u(T_1) = 0.1°C$$
$$u(T_2) = 0.15°C$$

$r_{12} = 0$ $\quad u(\Delta T) = 0.18°C$

$r_{12} = 1$ $\quad u(\Delta T) = 0.05°C$

$r_{12} = -1$ $\quad u(\Delta T) = 0.25°C$

Law of propagation of Uncertainties (2)

combined uncertainty of measurand *y*:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{x_j} u(x_i, x_j)$$

$$u_c^2(y) = C_f M_x C_f^T$$

$$C_f = \left(\frac{\partial f}{\partial x_1} \cdots\cdots \frac{\partial f}{\partial x_N}\right)^T$$

sensitivity coefficients

$$M_x = \begin{bmatrix} u^2(x_1) & \cdots & u(x_1, x_N) \\ \vdots & \ddots & \vdots \\ u(x_N, x_1) & \cdots & u^2(x_N) \end{bmatrix}$$

error covariance matrix

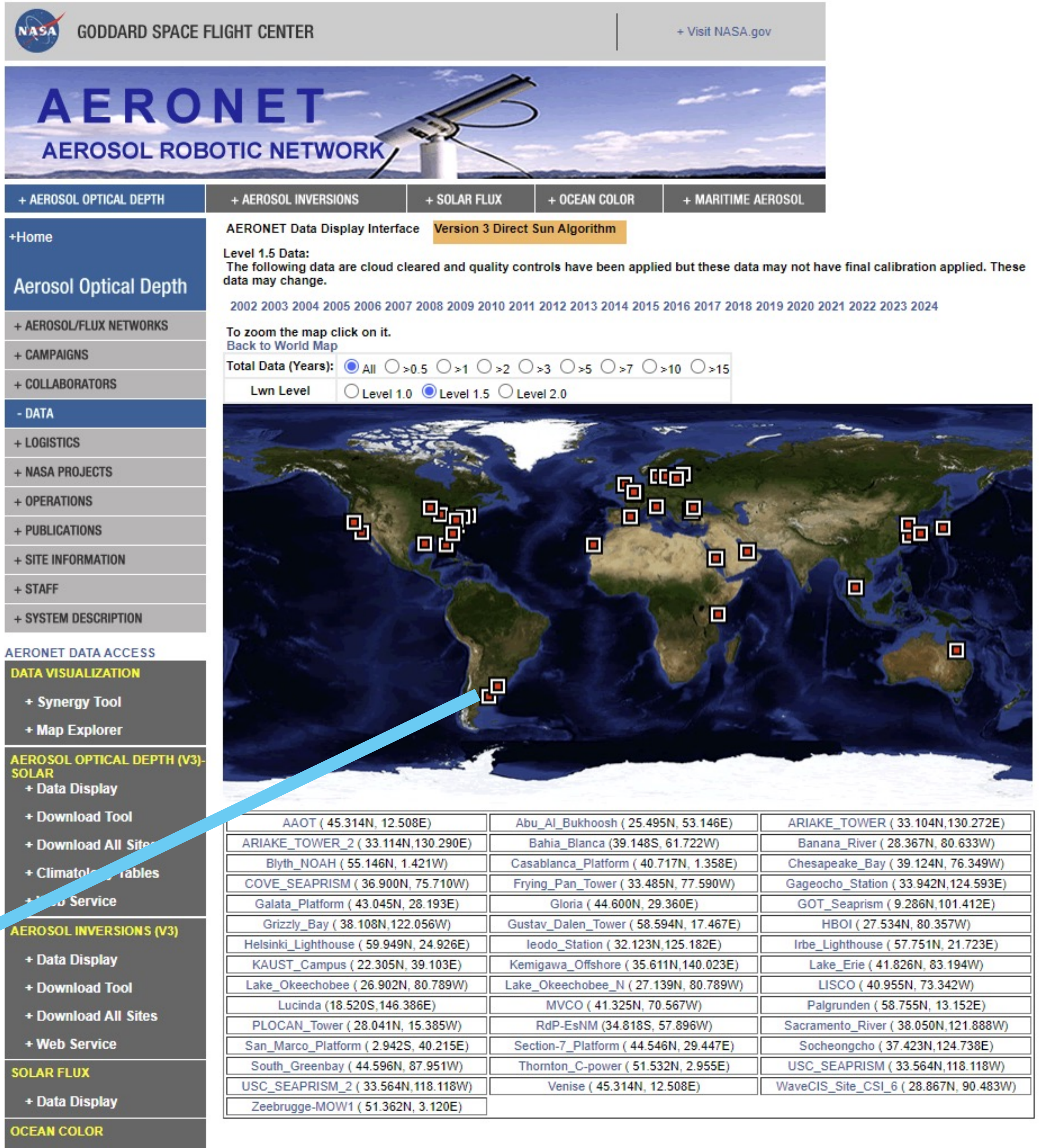
Application to Radiometric Data

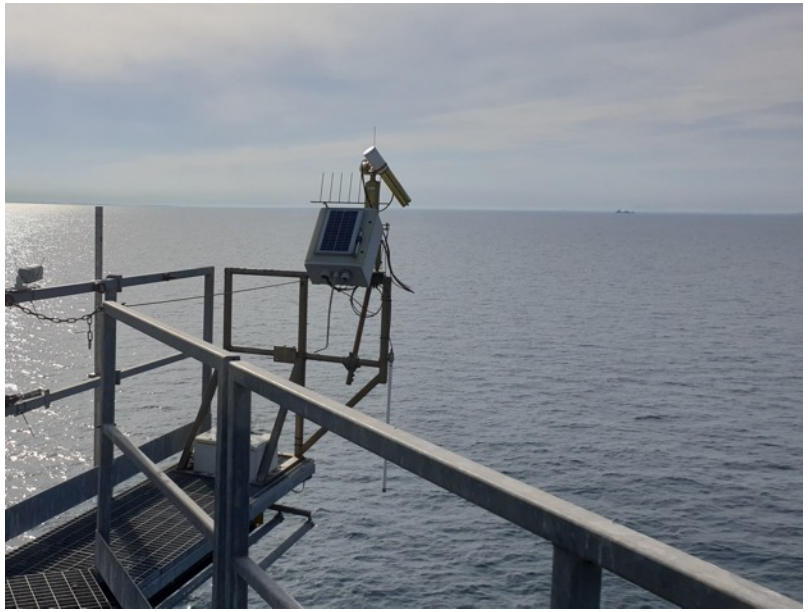
Uncertainties of AERONET-OC R_{RS} data

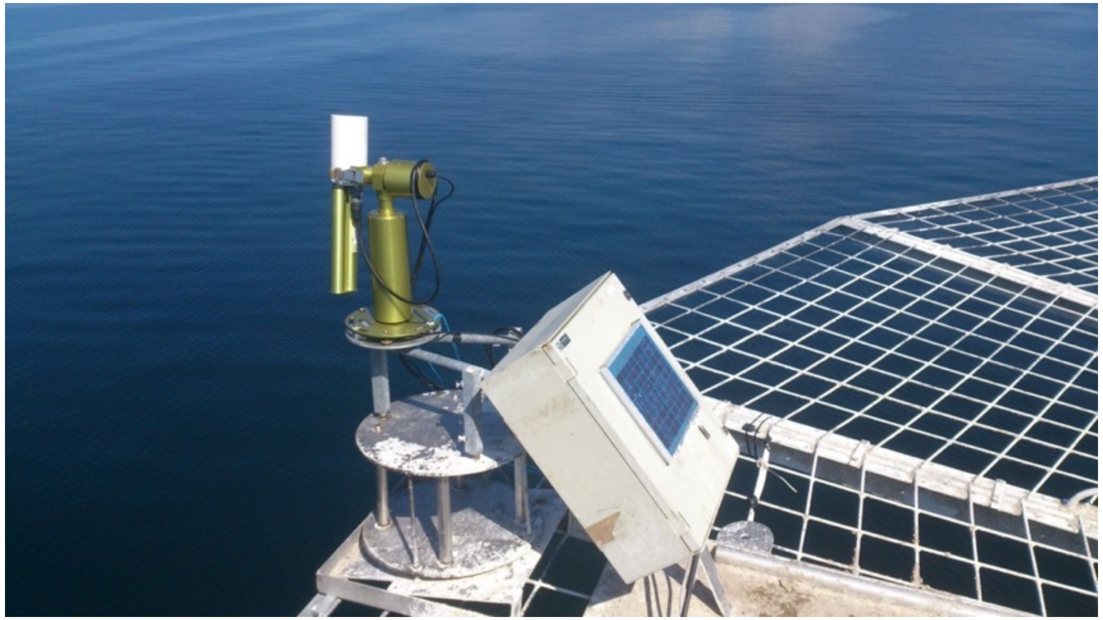
AERONET-OC

born as a JRC/NASA collaboration, generates globally distributed, highly consistent, time-series of standardized reflectance $R_{RS}(\lambda)$ and $\tau_a(\lambda)$ measurements to support Ocean Color validation activities

- Bahia_Blanca
- RdP-EsNM







N_T=11 measurements for L_T (average of lowest 2)
N_i=2 for L_i (average)

Gergely & Zibordi *M* 2014

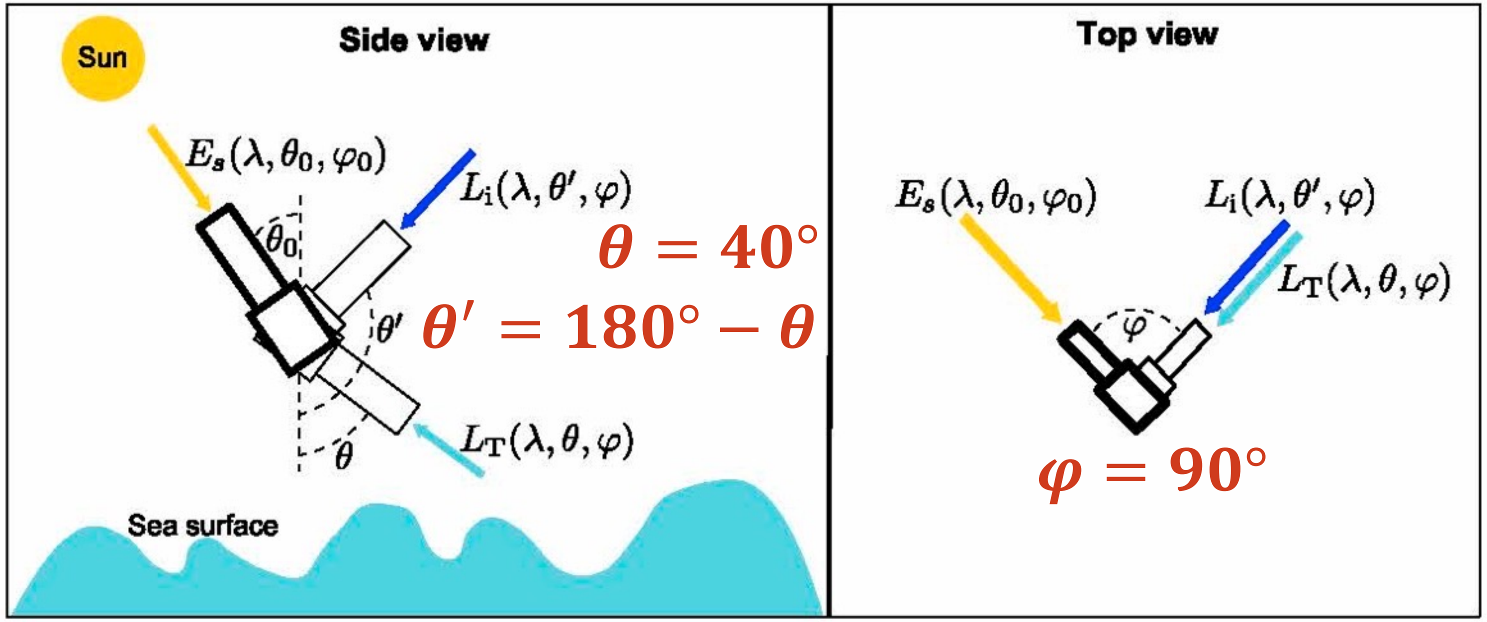
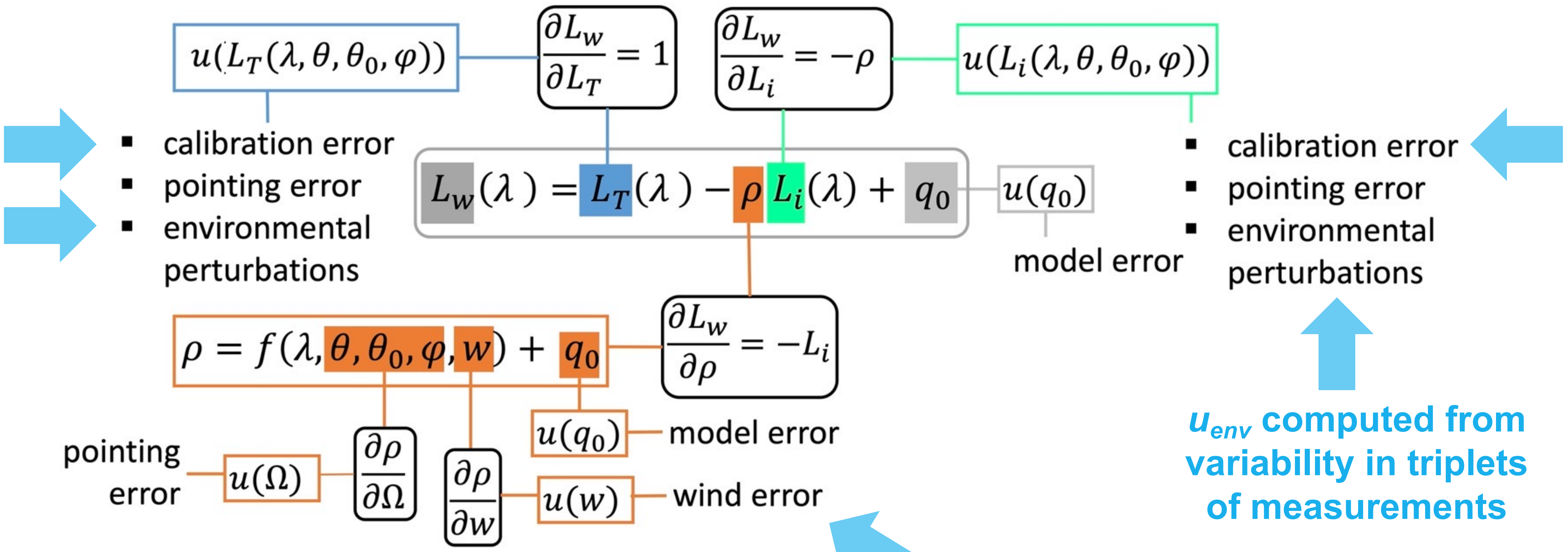


Figure 1. AERONET-OC measurement geometry for direct solar irradiance E_s, sky-radiance L_i, and total radiance from the sea L_T.

Measurement equation for L_w

Mélin et al. *FRS* 2024



***u_{env}* computed from variability in triplets of measurements**

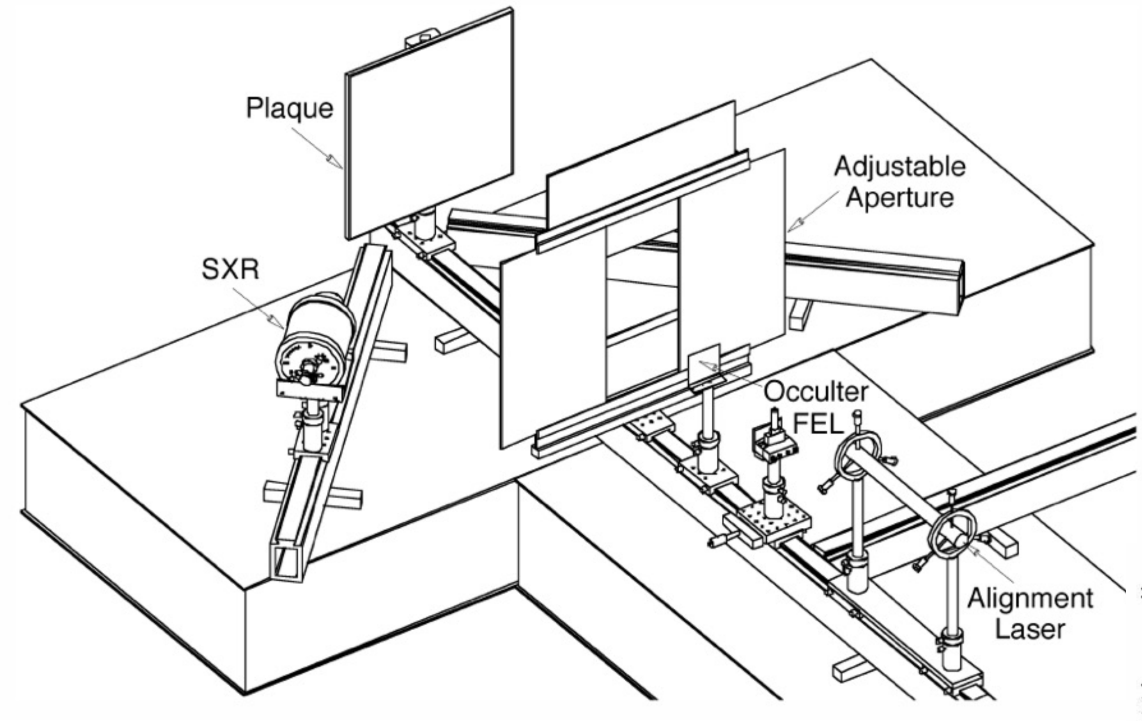
$u_{\%}(\rho)$ computed from values of *w*

Standard uncertainty for L_w:

$$u^2(L_w) = (L_T u_{\%}(L_T))^2 + (L_i u_{\%}(L_i)\rho)^2 + (L_i u_{\%}(\rho)\rho)^2$$

Cazzaniga & Zibordi *JAOT* 2023

Calibration



Hooker et al., *SeaWiFS Postlaunch NASA-TM*, vol. 17, 2002

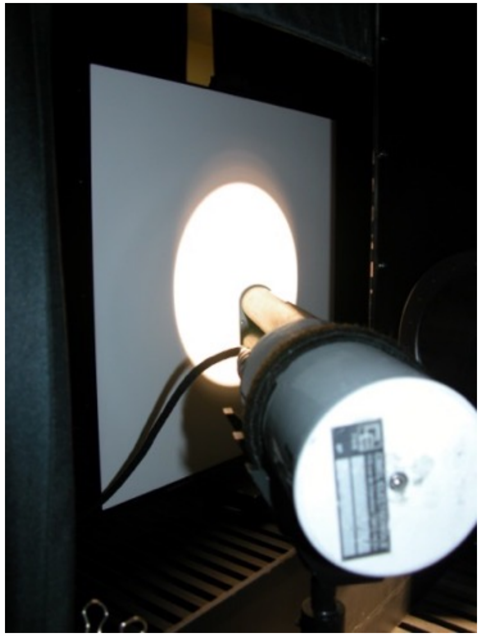


TABLE A1. Sources and related relative uncertainties (in %) contributing to absolute radiometric calibration uncertainties.

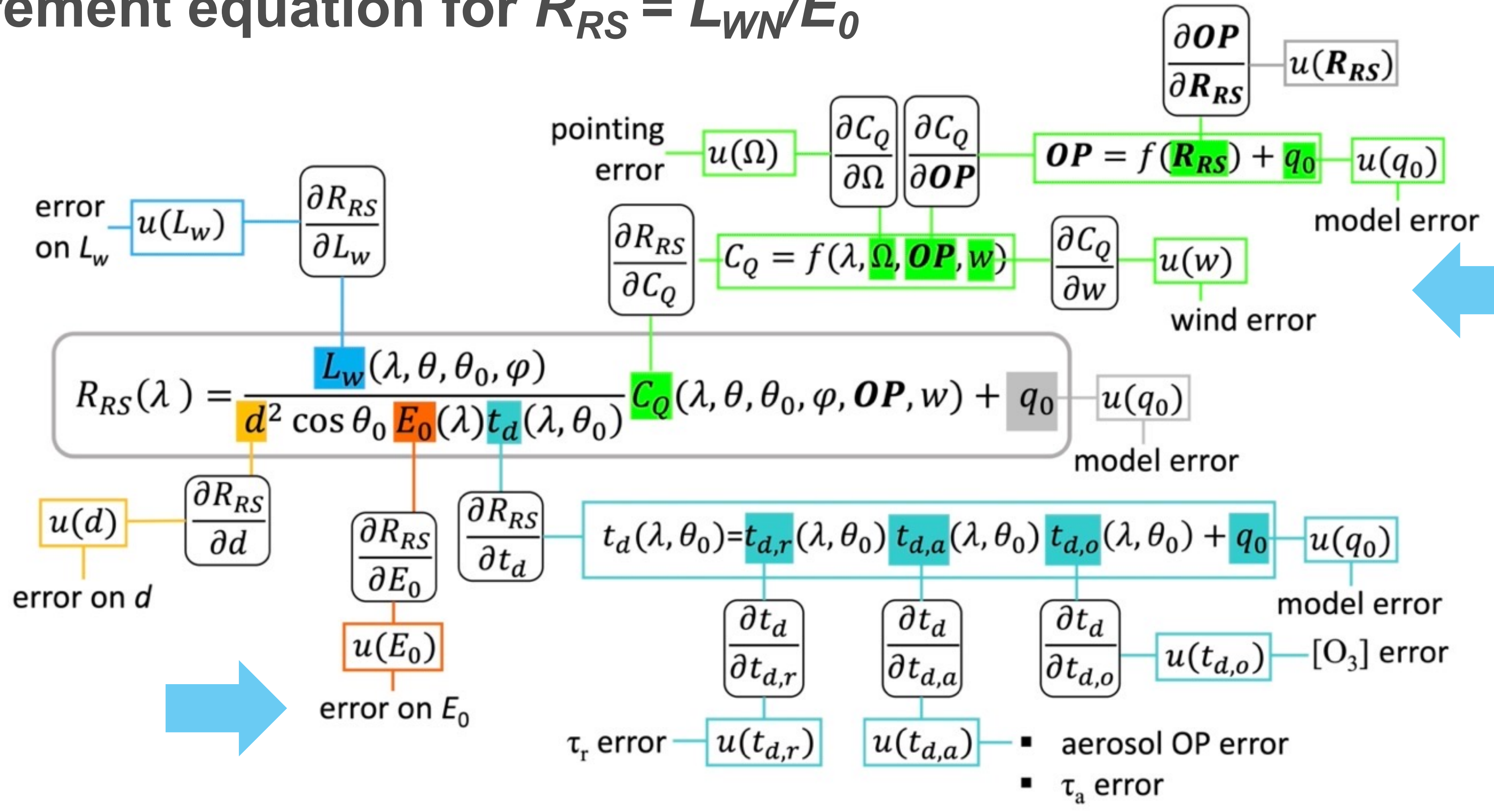
	λ (nm)							
	400	412	443	490	510	560	620	667
Lamp (NPL #1333)	0.90	0.55	0.55	0.50	0.50	0.45	0.45	0.40
Lamp fit	0.30	0.30	0.20	0.20	0.20	0.20	0.20	0.20
Lamp aging (with 25 h of use)	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Plaque	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Shunt	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Power supply (with a 7.5 mA bias)	0.07	0.07	0.07	0.06	0.06	0.05	0.05	0.04
Lamp–plaque distance	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Lamp positioning	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Plaque repositioning	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Sensor alignment	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10

Cazzaniga & Zibordi *JAOT* 2023

+ sensitivity decay

Measurement equation for $R_{RS} = L_{WN}/E_0$

Mélin et al. *FRS* 2024



$u_{\%}(C_Q)$ from Talone et al. (2018)

$$C_A = d^2 cos\theta_0 t_d$$

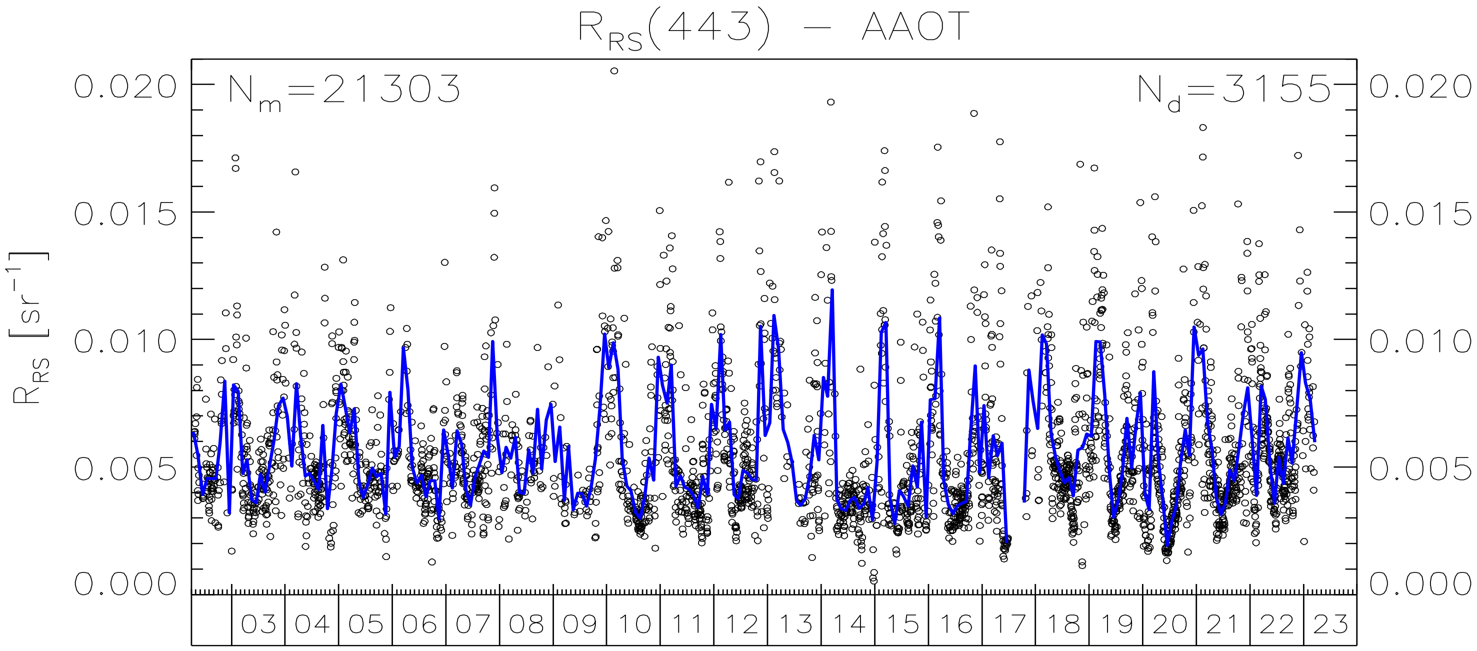
$u_{\%}(C_A)$ =1.5%

Standard uncertainty for L_{WN}:

$$u^2(L_{WN}) = (C_Q C_A u(L_w))^2 + (L_w u_{\%}(C_Q) C_A C_Q)^2 + (L_w C_Q C_A u_{\%}(C_A) C_A C_Q)^2$$

Cazzaniga & Zibordi *JAOT* 2023

AAOT: Acqua Oceanographic Tower



Sciuto et al. 2023

Uncertainty at **AAOT**:

Source of uncertainty	$u_{\%}(L_{WN})$				
	412	443	488	551	667
absolute calibration	2.7	2.7	2.7	2.7	2.7
sensitivity change	0.4	0.2	0.2	0.2	0.2
correction C_Q	1.6	2.0	2.8	2.9	1.9
t_d	1.5	1.5	1.5	1.5	1.5
ρ	1.8	1.3	0.7	0.6	2.5
wind *w*	1.1	0.8	0.4	0.4	0.4
environmental effects	3.1	2.1	2.1	2.1	6.4
quadrature sum	**5.1**	**4.5**	**4.7**	**4.7**	**7.8**

Zibordi et al. *JAOT* 2009

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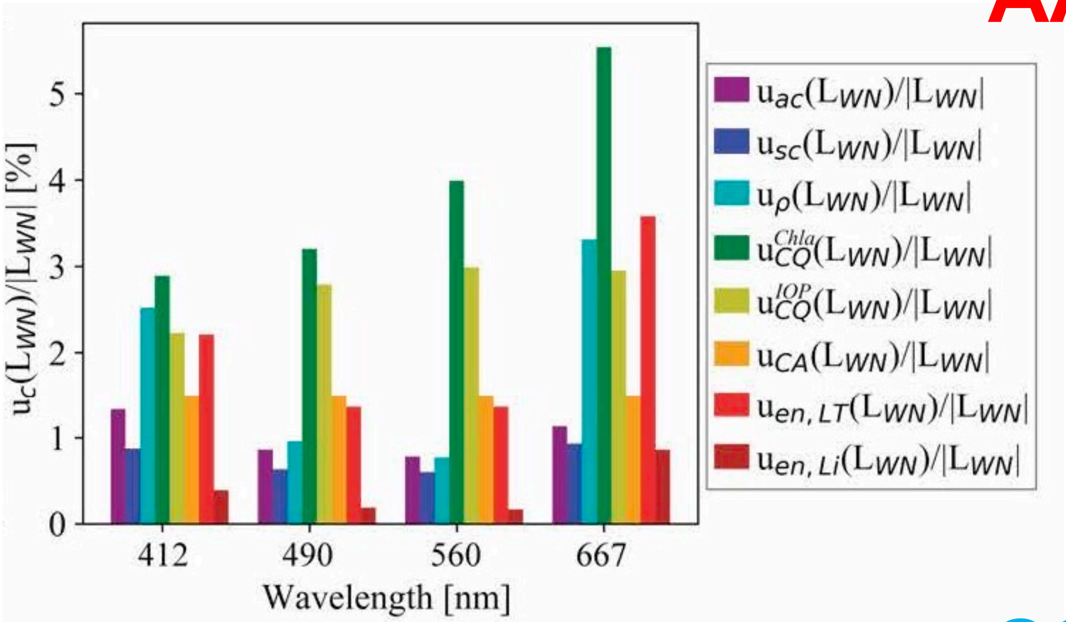
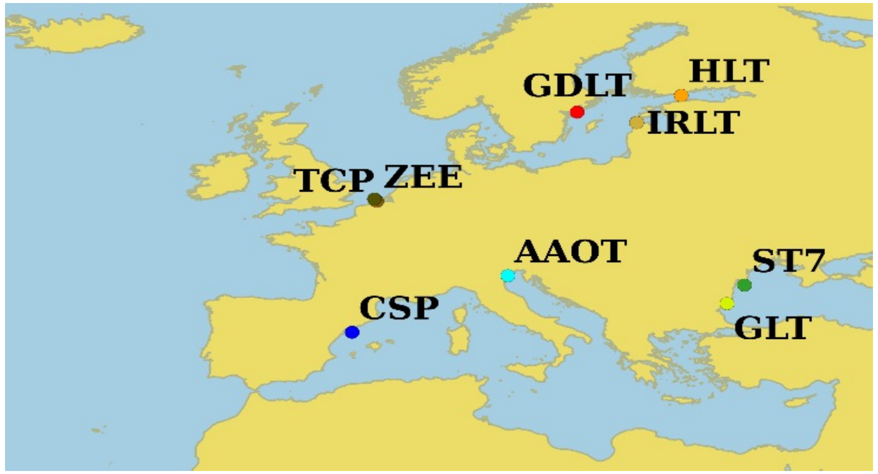
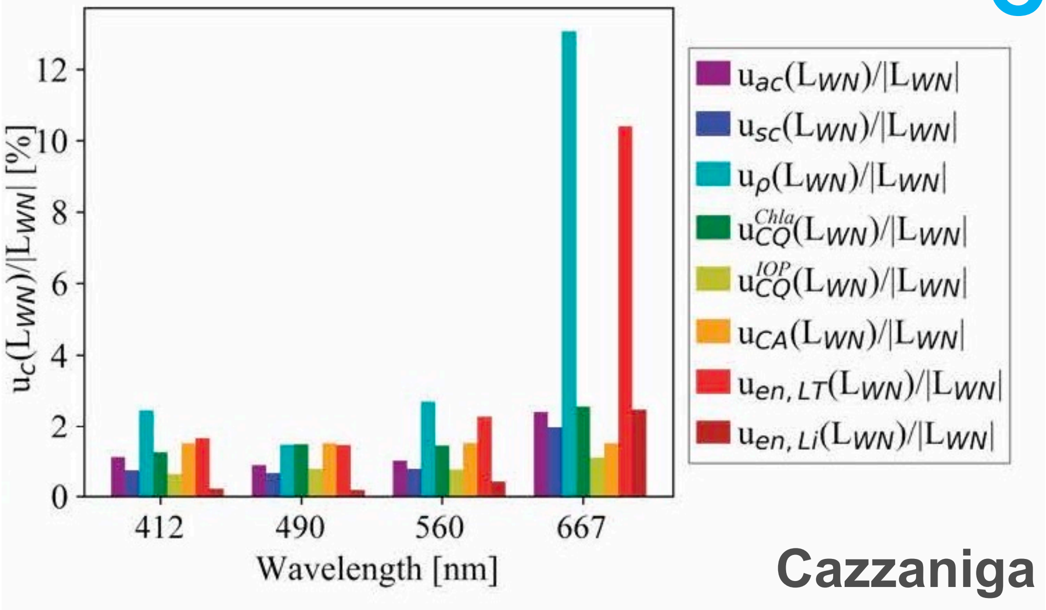
Uncertainty budget at : AAOT





Cazzaniga & Mélin 2024

CSP: Casablanca Platform



NB: budget variable with water type

Cazzaniga & Zibordi *JAOT* 2023

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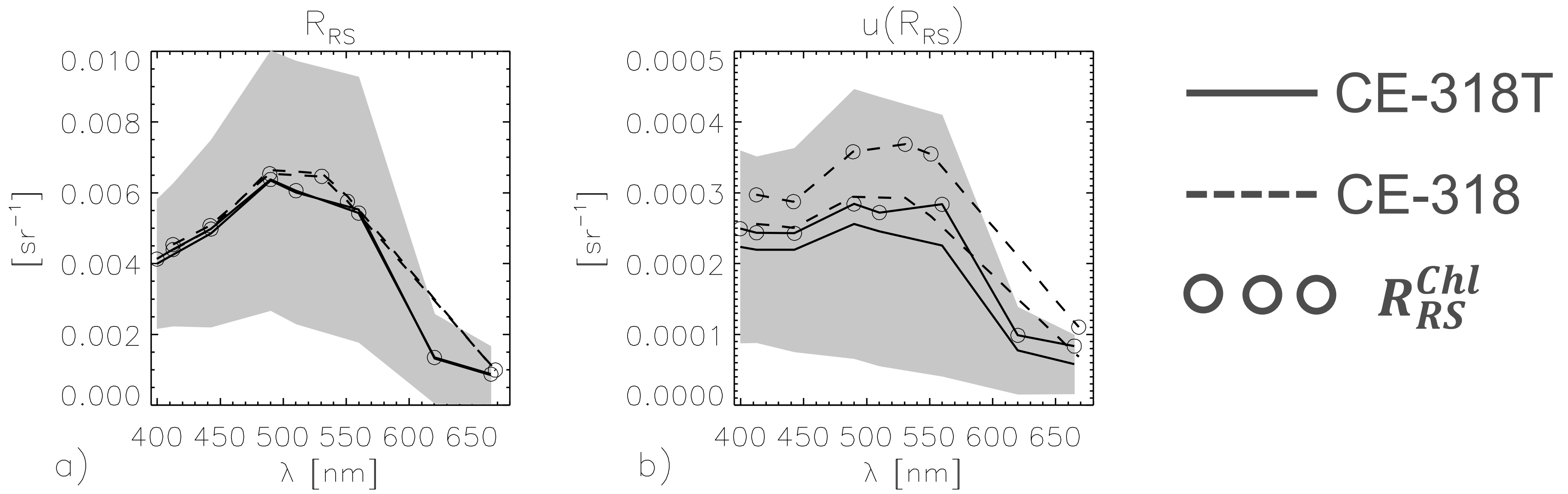
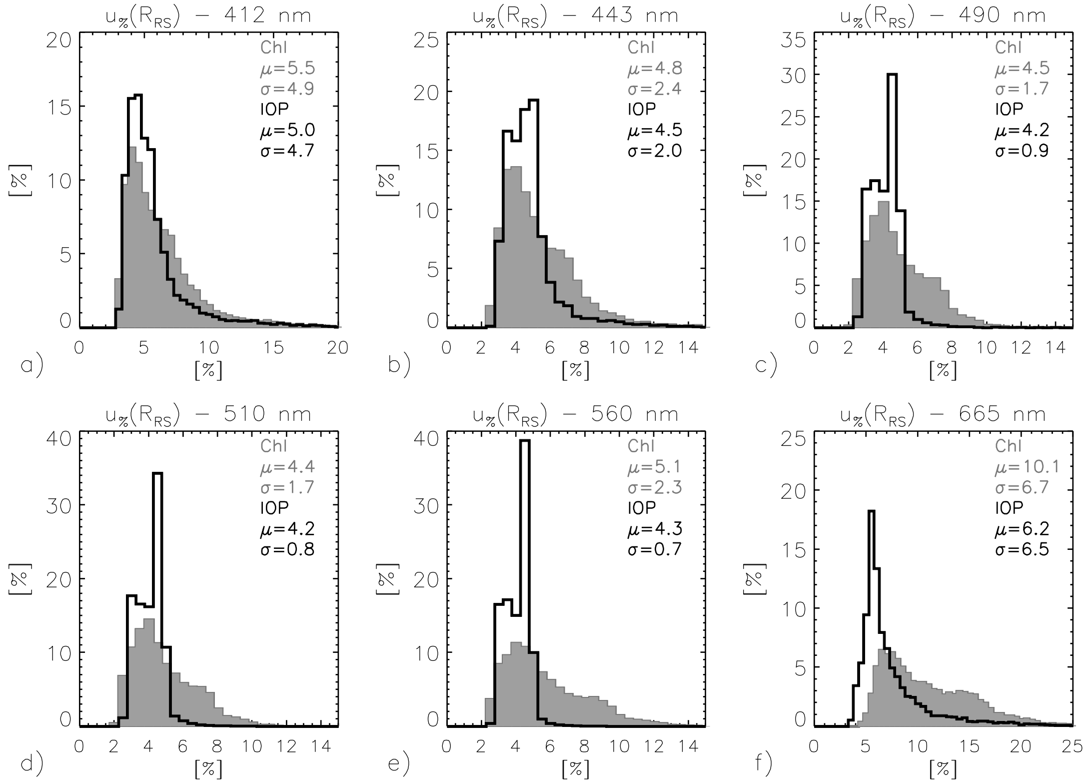
AAOT



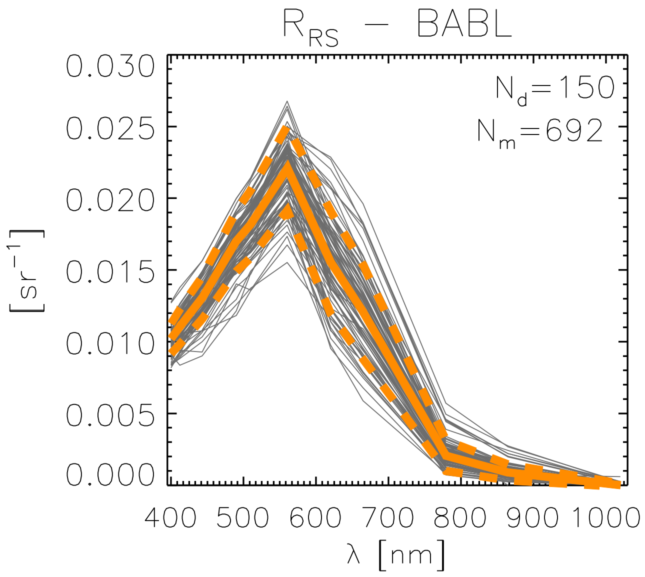
$u_{\%}(R_{RS})$

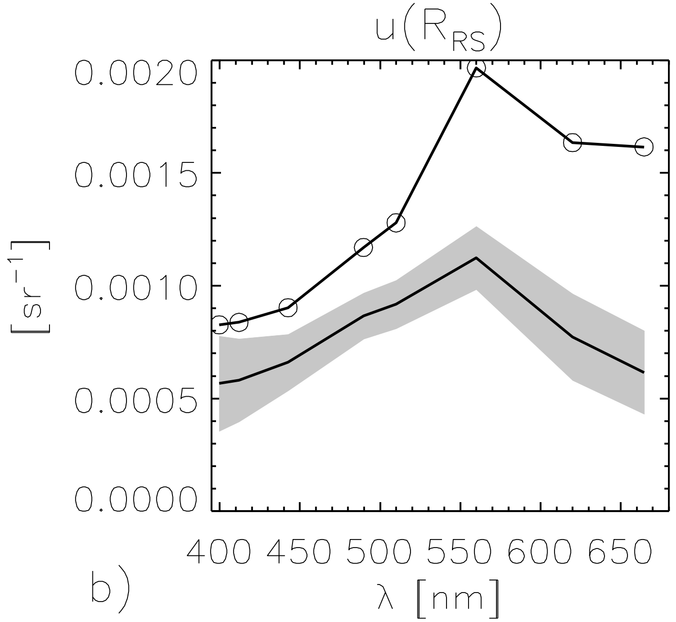


Mélin et al. *FRS* 2024

Argentinian sites (tentative results):

Bahia Blanca

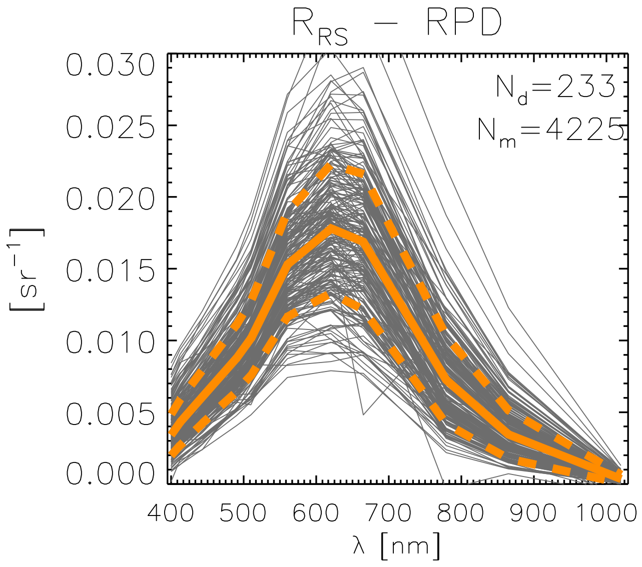


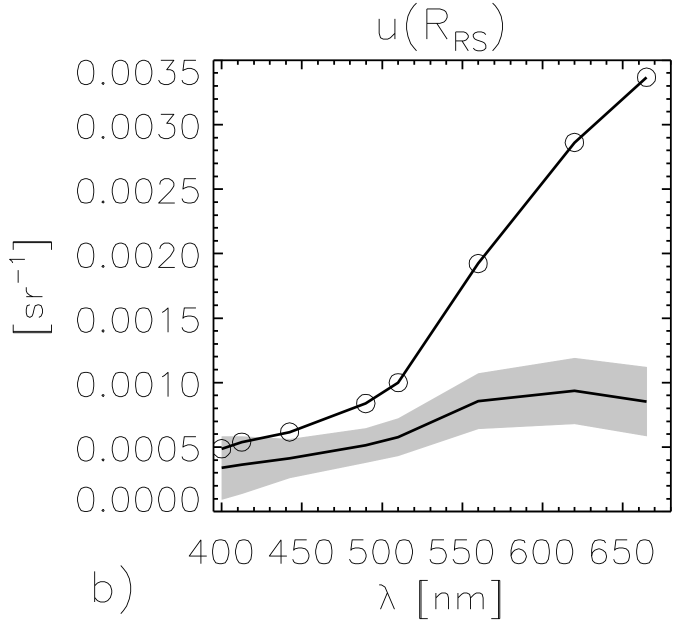


C_Q: Chl-based

C_Q: IOP-based

Rio de la Plata

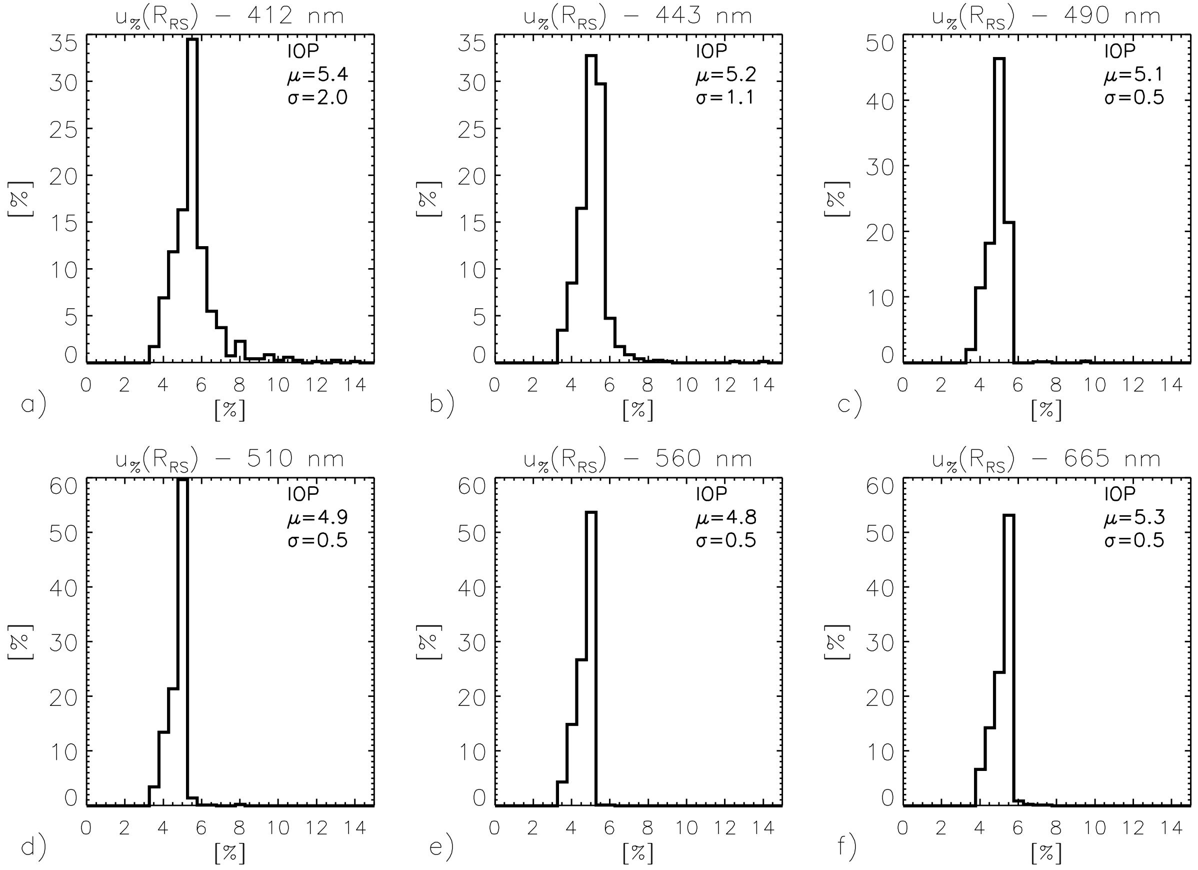




NB: only Level 1.5 data

Argentinian sites (tentative results): $u_{\%}(R_{RS})$

Bahia Blanca



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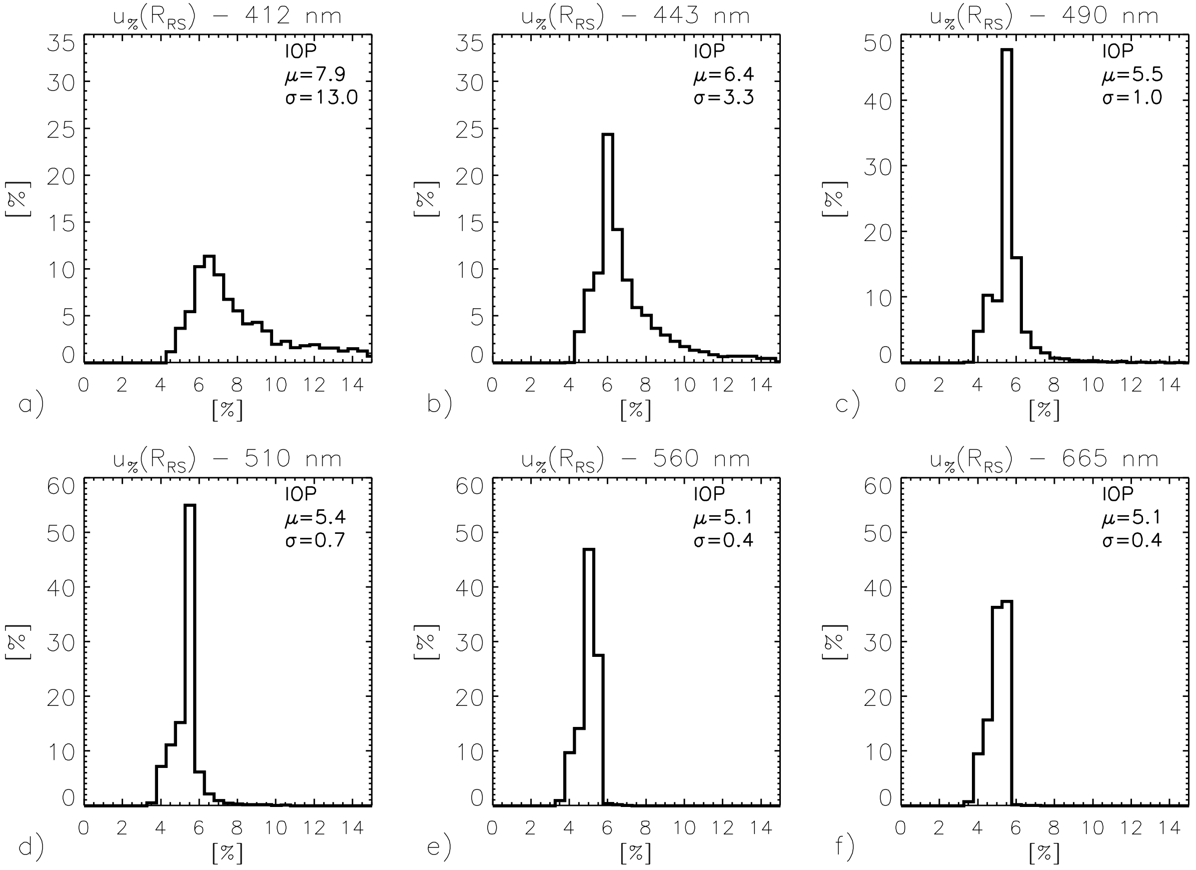

Courtesy: I. Cazzaniga

- ✓ A measurement is meaningless unless accompanied by a statement of uncertainty
- ✓ Respect terminology
- ✓ Include metrological practices right at the start of any project