

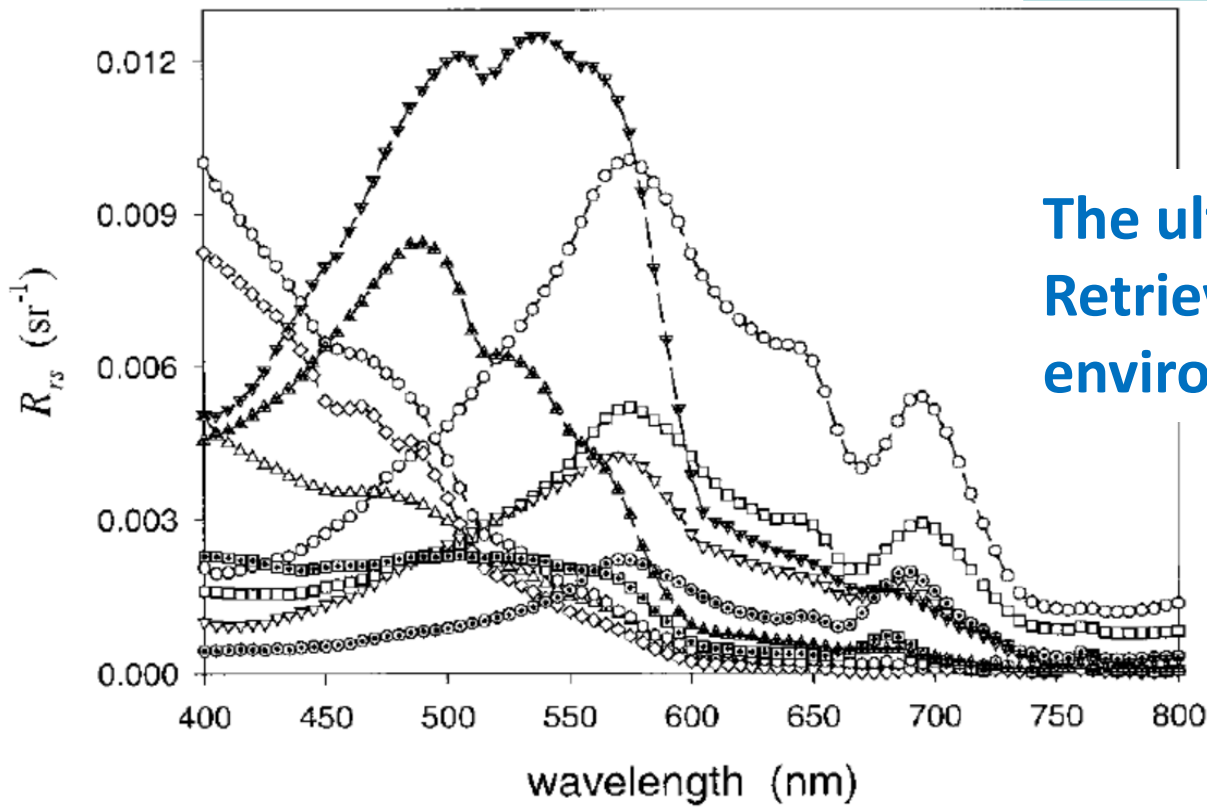
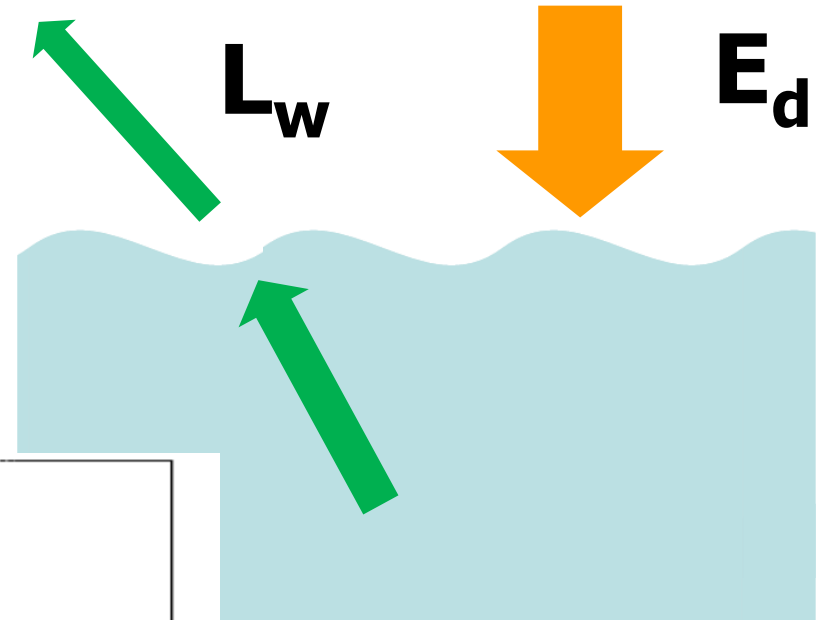
Inherent Optical Properties (IOPs)

Lecture 2: Inversion



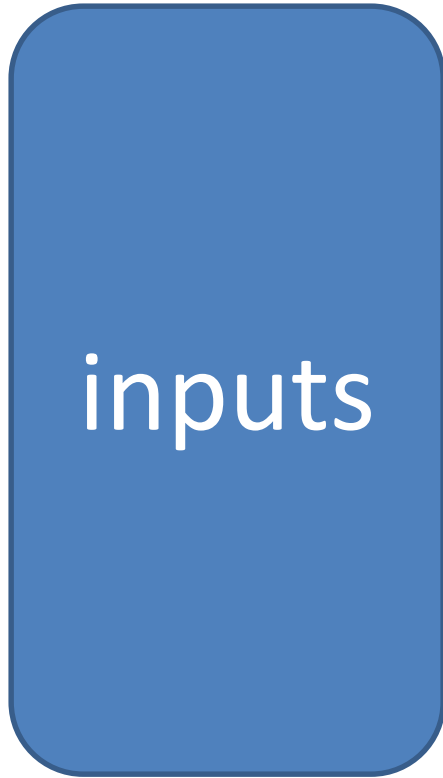
Remote-sensing reflectance (sr^{-1}):

$$R_{rs}(\lambda) = \frac{L_w(\lambda, 0^+)}{E_d(\lambda, 0^+)}$$



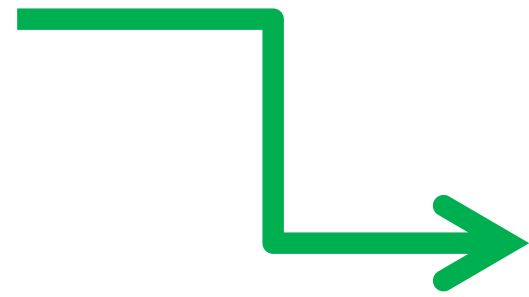
The ultimate objective of RS:
Retrieval useful/important
environmental information

How?
algorithm!



$(L_w \text{ or } R_{rs})$

algorithm



(IOPs or
[Chl] etc)

Empirical
(explicit or implicit)

Bio-optical models: **No need**

Semi-analytical
(algebraic, LUT, optimization)

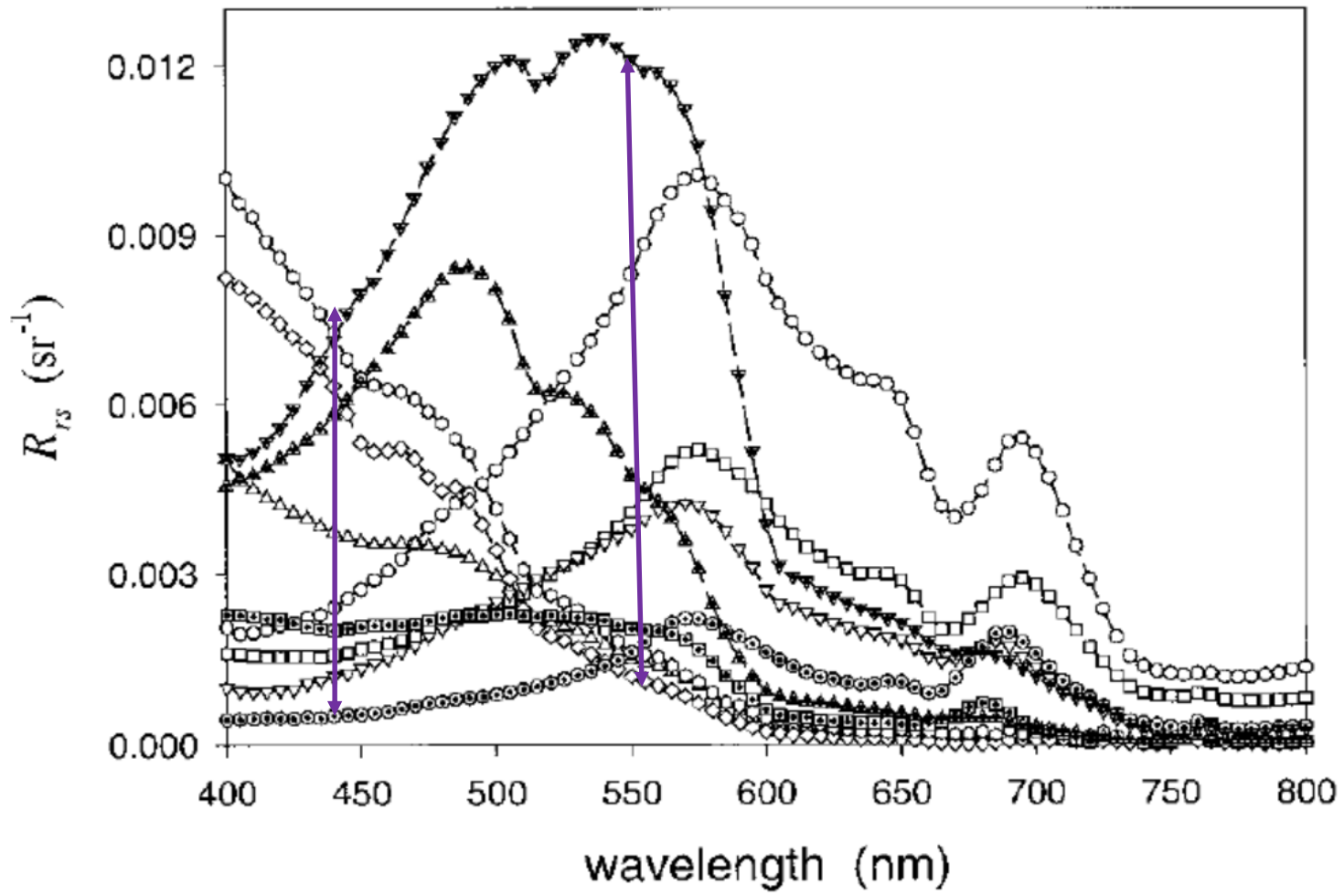
Bio-optical models: **Yes**

Bottom Up Strategy (BUS)
Top Down Strategy (TDS)

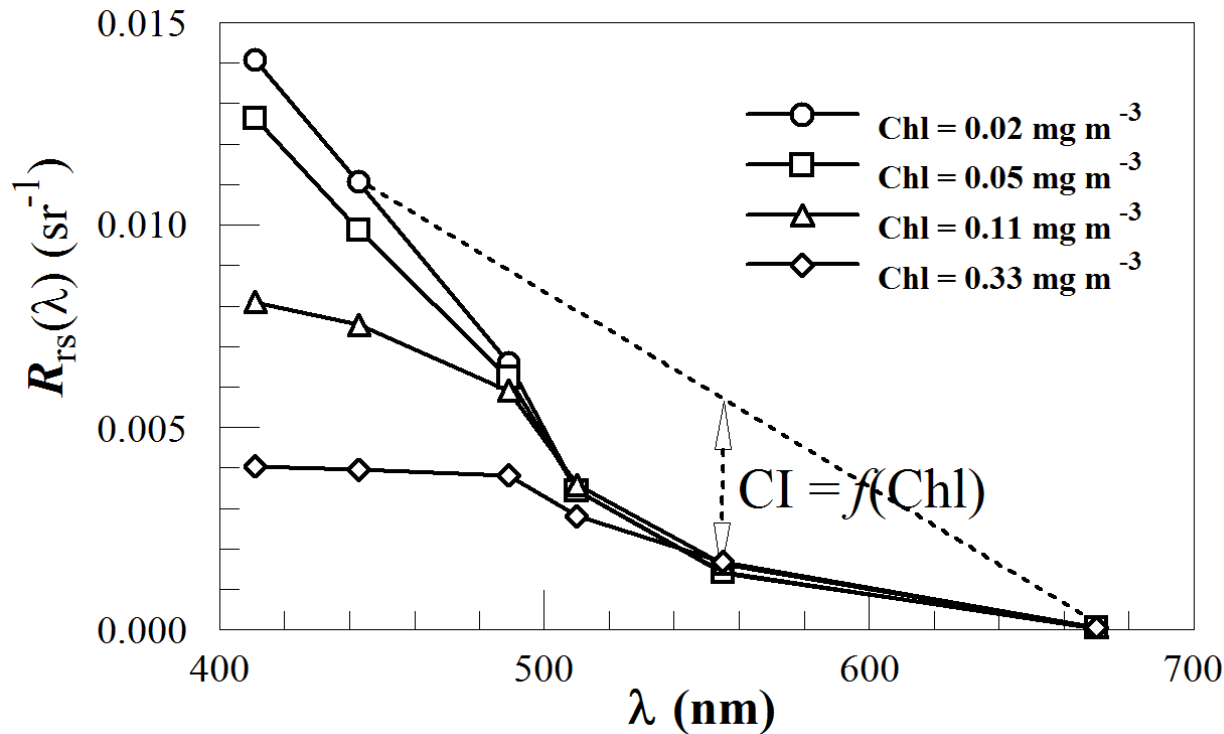
Chl centered band-ratio algorithms

Algorithm	Type	Result Equation(s)	Band Ratio (R), Coefficients (a)
Global processing (GPs)	power	$C_{13} = 10^{(a0+a1*R1)}$ $C_{23} = 10^{(a2+a3*R2)}$ $[C + P] = C_{13}$; if C_{13} and $C_{23} > 1.5 \mu\text{g L}^{-1}$ then $[C + P] = C_{23}$	$R1 = \log(\text{Lwn443}/\text{Lwn550})$ $R2 = \log(\text{Lwn520}/\text{Lwn550})$ $a = [0.053, 1.705, 0.522, 2.440]$
Clark three-band (C3b)	power	$[C + P] = 10^{(a0+a1*R)}$	$R = \log((\text{Lwn443} + \text{Lwn520})/\text{Lwn550})$ $a = [0.745, -2.252]$
Aiken-C	hyperbolic + power	$C_{21} = \exp(a0 + a1*\ln(R))$ $C_{23} = (R + a2)/(a3 + a4*R)$ $C = C_{21}$; if $C < 2.0 \mu\text{g L}^{-1}$ then $C = C_{23}$	$R = \text{Lwn490}/\text{Lwn555}$ $a = [0.464, -1.989, -5.29, 0.719, -4.23]$
Aiken-P	hyperbolic + power	$C_{22} = \exp(a0 + a1*\ln(R))$ $C_{24} = (R + a2)/(a3 + a4*R)$ $[C + P] = C_{22}$; if $[C + P] < 2.0 \mu\text{g L}^{-1}$ then $[C + P] = C_{24}$	$R = \text{Lwn490}/\text{Lwn555}$ $a = [0.696, -2.085, -5.29, 0.592, -3.48]$
OCTS-C	power	$C = 10^{(a0+a1*R)}$	$R = \log((\text{Lwn520} + \text{Lwn565})/\text{Lwn490})$ $a = [0.55006, 3.497]$
OCTS-P	multiple regression	$[C + P] = 10^{(a0+a1*R1+a2*R2)}$	$R1 = \log(\text{Lwn443}/\text{Lwn520})$ $R2 = \log(\text{Lwn190}/\text{Lwn520})$ $a = [0.19535, -2.079, -3.497]$
POLDER	cubic	$C = 10^{(a0+a1*R+a2*R^2+a3*R^3)}$	$R = \log(\text{Rrs443}/\text{Rrs565})$ $a = [0.438, -2.114, 0.916, -0.851]$
CalCOFI two-band linear	power	$C = 10^{(a0+a1*R)}$	$R = \log(\text{Rrs490}/\text{Rrs555})$ $a = [0.444, -2.431]$
CalCOFI two-band cubic	cubic	$C = 10^{(a0+a1*R+a2*R^2+a3*R^3)}$	$R = \log(\text{Rrs490}/\text{Rrs555})$ $a = [0.450, -2.860, 0.996, -0.3674]$
CalCOFI three-band	multiple regression	$C = \exp(a0 + a1*R1 + a2*R2)$	$R1 = \ln(\text{Rrs490}/\text{Rrs555})$ $R2 = \ln(\text{Rrs510}/\text{Rrs555})$ $a = [1.025, -1.622, 1.238]$
CalCOFI four-band	multiple regression	$C = \exp(a0 + a1*R1 + a2*R2)$	$R1 = \ln(\text{Rrs443}/\text{Rrs555})$ $R2 = \ln(\text{Rrs412}/\text{Rrs510})$ $a = [0.753, -2.583, 1.389]$
Morel-1	power	$C = 10^{(a0+a1*R)}$	$R = \log(\text{Rrs443}/\text{Rrs555})$ $a = [0.2492, -1.768]$
Morel-2	power	$C = \exp(a0 + a1*R)$	$R = \ln(\text{Rrs490}/\text{Rrs555})$ $a = [1.077835, -2.542605]$
Morel-3	cubic	$C = 10^{(a0+a1*R+a2*R^2+a3*R^3)}$	$R = \log(\text{Rrs443}/\text{Rrs555})$ $a = [0.20766, -1.82878, 0.75885, -0.73979]$
Morel-4	cubic	$C = 10^{(a0+a1*R+a2*R^2+a3*R^3)}$	$R = \log(\text{Rrs490}/\text{Rrs555})$ $a = [1.03117, -2.40134, 0.3219897, -0.291066]$

(O'Reilly et al 1998)

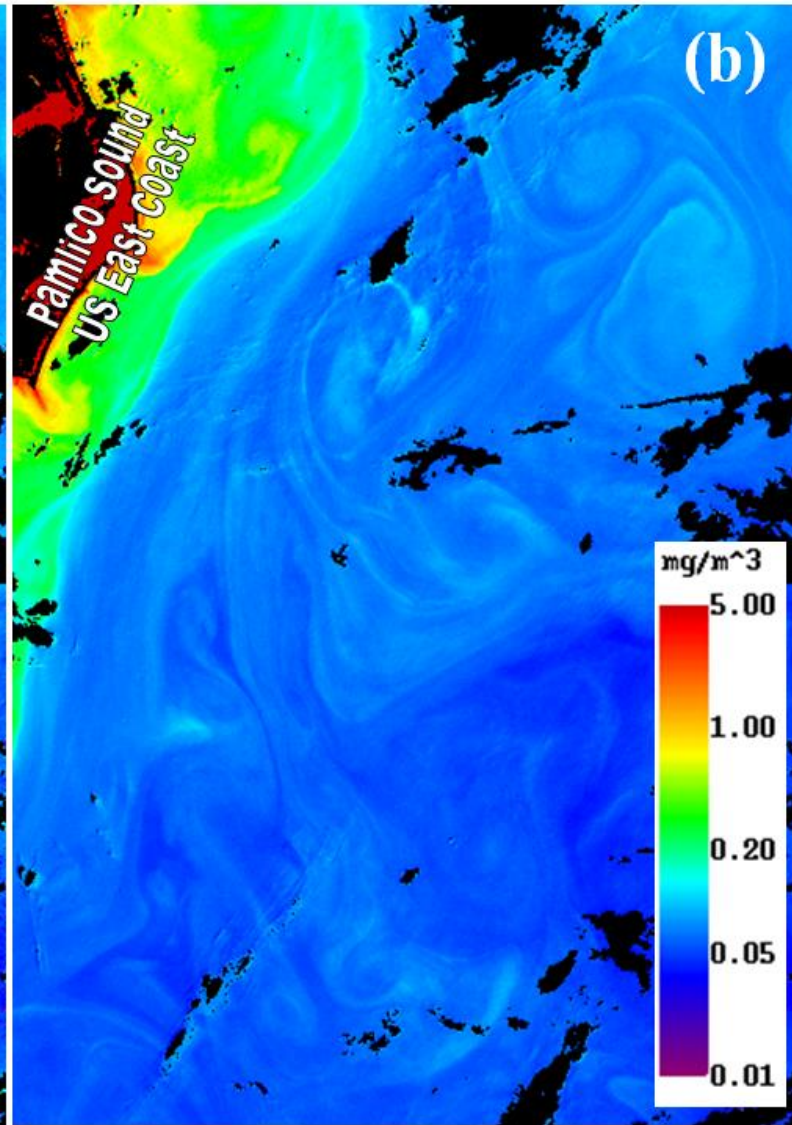
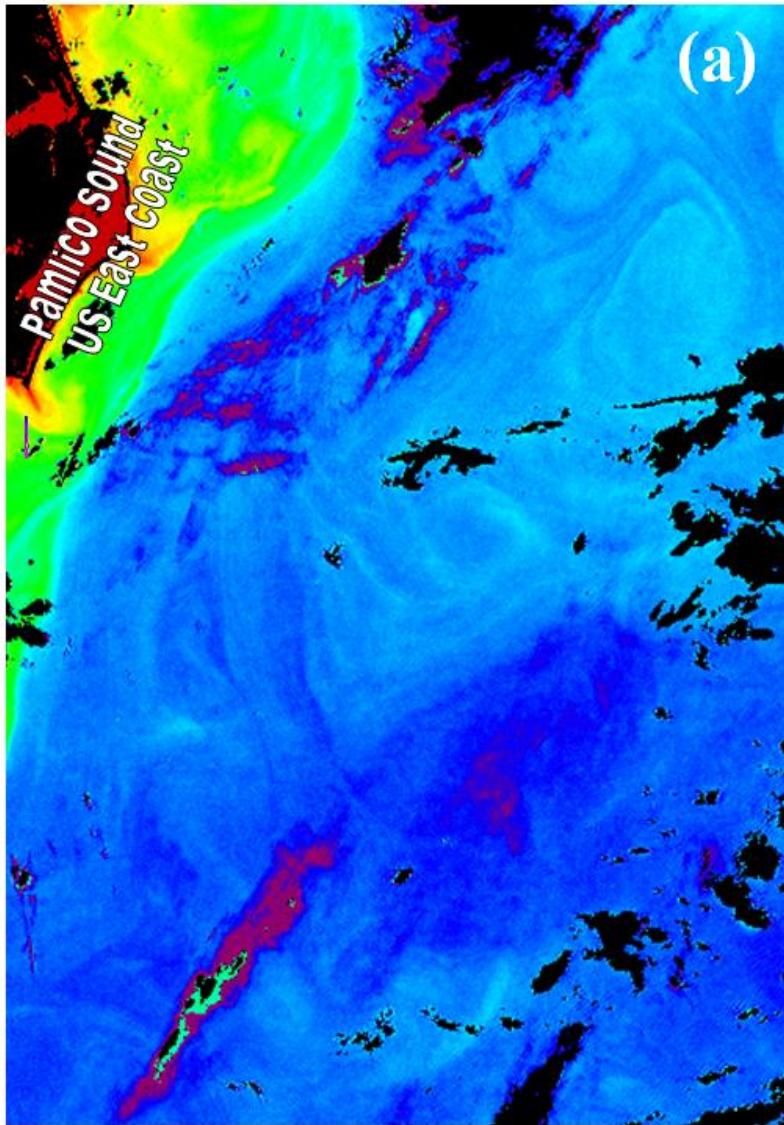


Chl based on band difference

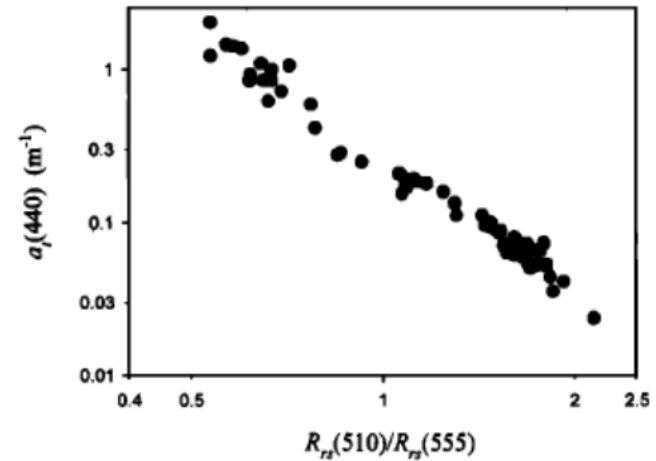
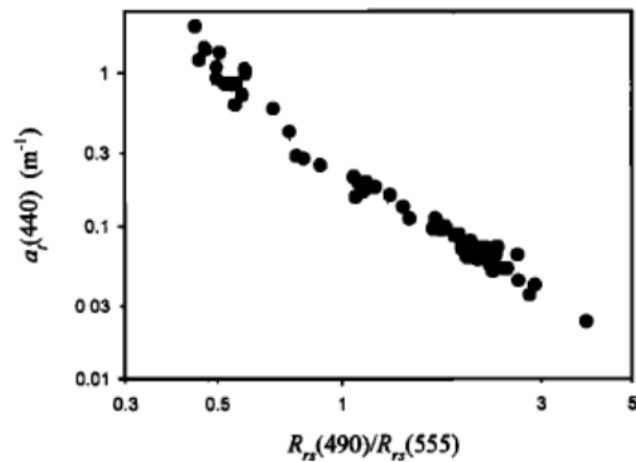
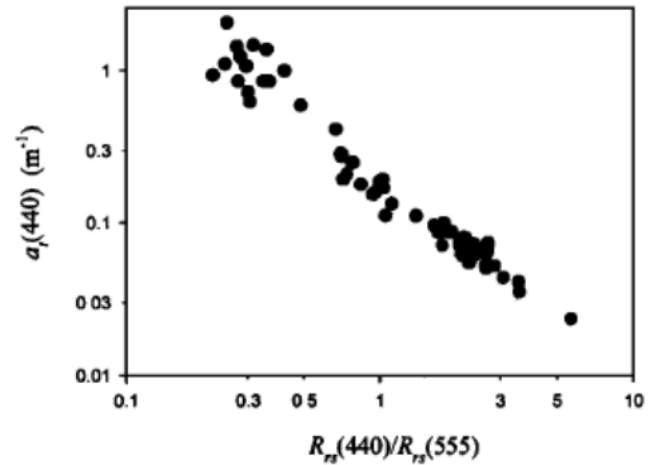
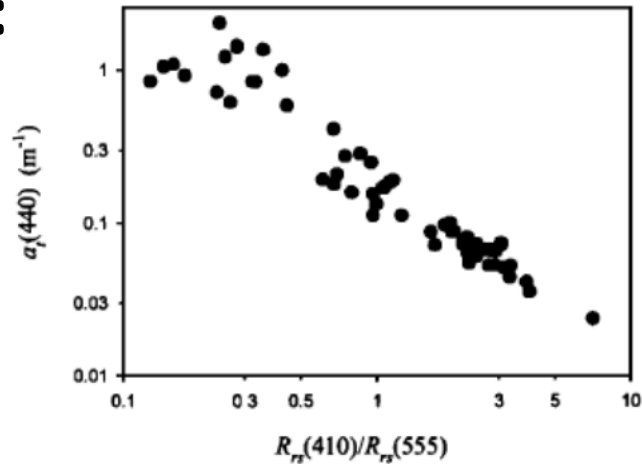


$$CI = R_{rs}(555) - \left\{ R_{rs}(443) + \frac{555 - 443}{670 - 443} \times [R_{rs}(670) - R_{rs}(443)] \right\}$$

$$Chl = 10^{-0.49 + 191.66 CI}; \quad CI \leq -0.0005 \text{ sr}^{-1}$$



Empirical:



$$a_i(440) = 10^{-0.674 - 0.531\rho_{25} - 0.745\rho_{25}^2 - 1.469\rho_{35} + 2.375\rho_{35}^2}, \quad (14)$$

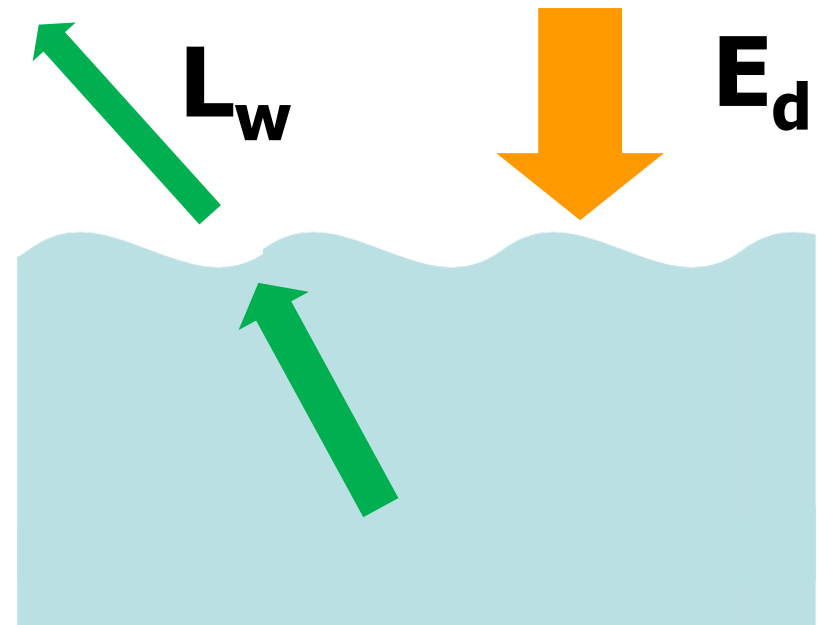
$$a_\phi(440) = 10^{-0.919 + 1.037\rho_{25} - 0.407\rho_{25}^2 - 3.531\rho_{35} + 1.579\rho_{35}^2}, \quad (20)$$

(Lee et al 1998)

Physics-based algorithms (mechanistic)

Remote-sensing reflectance (sr^{-1}):

$$R_{rs}(\lambda) = \frac{L_w(\lambda, 0^+)}{E_d(\lambda, 0^+)}$$



How is R_{rs} related to water's optical (biogeochemical) properties?

Physics-based algorithms (mechanistic)

How is R_{rs} related to water's optical (biogeochemical) properties?

Radiative Transfer Equation (no inelastic scattering):

$$\frac{d L(\Omega)}{d l} = -c L(\Omega) + \int L(\Omega') \beta(\Omega', \Omega) d\omega$$

$$\frac{d L_u(\pi, z)}{d z} = -c L_u(\pi, z) + \int L(\Omega') \beta(\Omega', \Omega) d\omega$$

Exact solution (no inelastic scattering): $r_{rs}(\lambda, \Omega') = \frac{L_u(0^-, \Omega')}{E_d(0^-)}$

$$r_{rs}(\lambda, \Omega') \equiv \frac{D_d(\lambda, \theta_S')}{c(\lambda) + k_L(\lambda, \Omega') - f_L(\lambda, \Omega')b_f(\lambda)} \frac{\int_0^{2\pi} \int_0^{\pi/2} \beta(\Omega', \Omega) L(\lambda, \Omega') \sin(\theta') d\theta' d\varphi'}{E_{od}(0^-, \lambda, \theta_S')} \quad (\text{Zaneveld 1995})$$

Albert and Mobley (2003) :

$$r_{rs}(\lambda, \Omega') = q(\Omega', w) \sum_{i=1}^4 p_i \left(\frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)} \right)^i$$

Lee et al (2004)

$$r_{rs}(\lambda, \Omega') = g_w(\Omega') \frac{b_{bw}(\lambda)}{a(\lambda) + b_b(\lambda)} + g_p(\lambda, \Omega') \frac{b_{bp}(\lambda)}{a(\lambda) + b_b(\lambda)}$$

Park and Ruddick (2005)

$$r_{rs}(\lambda, \Omega') = \sum_{i=1}^4 g_i(\Omega', \nu_b) \left(\frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)} \right)^i$$

Van Der Woerd and Pasterkamp (2008)

$$\ln[r_{rs}(\lambda, \Omega')] = \sum_{i=1}^4 \sum_{j=1}^4 P_{ij}(\Omega') [\ln(a(\lambda))]^i [\ln(b(\lambda))]^j$$

Morel et al (1993, 1996, 2002):

$$r_{rs}(\lambda, \Omega') = g(\lambda, Chl, \Omega') \frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)}$$

Gordon et al (1988):

$$r_{rs}(\lambda, \pi) = \sum_{i=1}^2 g_i \left(\frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)} \right)^i;$$

$$g_1 = 0.0949, g_2 = 0.0794$$

$r_{rs} \rightarrow R_{rs} ?$

$$R_{rs} = \frac{L_w}{E_d(0^+)}$$

$$E_d(0^-) = t_E E_d(0^+) + \gamma E_u(0^-)$$

$$L_w = \frac{t_L}{n_w} L_u(0^-)$$

$$R_{rs} = \frac{t_E t_L}{n_w^2} \frac{r_{rs}}{1 - \gamma R} = \frac{0.52 r_{rs}}{1 - 1.7 r_{rs}}$$

Solve Rrs for IOPs or in-water constituents?

Two basic strategies:

1. Bottom-up strategy (BUS):

Assume we know the spectral shapes of the optically active components

2. Top-down strategy (TDS):

Only need the spectral shape information when it is necessary

What are we facing in RS algorithms?

$$R_{rs}(\lambda) = F(a(\lambda), b_b(\lambda))$$



$$R_{rs}(\lambda) = F(a_w(\lambda), a_{ph}(\lambda), a_{dg}(\lambda), b_{bw}(\lambda), b_{bp}(\lambda))$$



$$\left\{ \begin{array}{l} R_{rs}(\lambda_1) = F(a_w(\lambda_1), a_{ph}(\lambda_1), a_{dg}(\lambda_1), b_{bw}(\lambda_1), b_{bp}(\lambda_1)) \\ R_{rs}(\lambda_2) = F(a_w(\lambda_2), a_{ph}(\lambda_2), a_{dg}(\lambda_2), b_{bw}(\lambda_2), b_{bp}(\lambda_2)) \\ \vdots \\ R_{rs}(\lambda_n) = F(a_w(\lambda_n), a_{ph}(\lambda_n), a_{dg}(\lambda_n), b_{bw}(\lambda_n), b_{bp}(\lambda_n)) \end{array} \right.$$

of unknowns > # of equations!

An ill formulated math problem!

Have to increase # of equations or decrease # of unknowns!

1. Bottom-up strategy (BUS):

Build-up an Rrs spectrum block-by-block:

$$a(\lambda) = a_w(\lambda) + \sum a_{xi}(\lambda) \quad b_b(\lambda) = b_{bw}(\lambda) + \sum b_{bxi}(\lambda)$$

$$a(\lambda) = a_w(\lambda) + a_{ph}(\lambda) + a_d(\lambda) + a_g(\lambda)$$

$$a(\lambda) = a_w(\lambda) + a_{ph}(\lambda) + a_{dg}(\lambda)$$

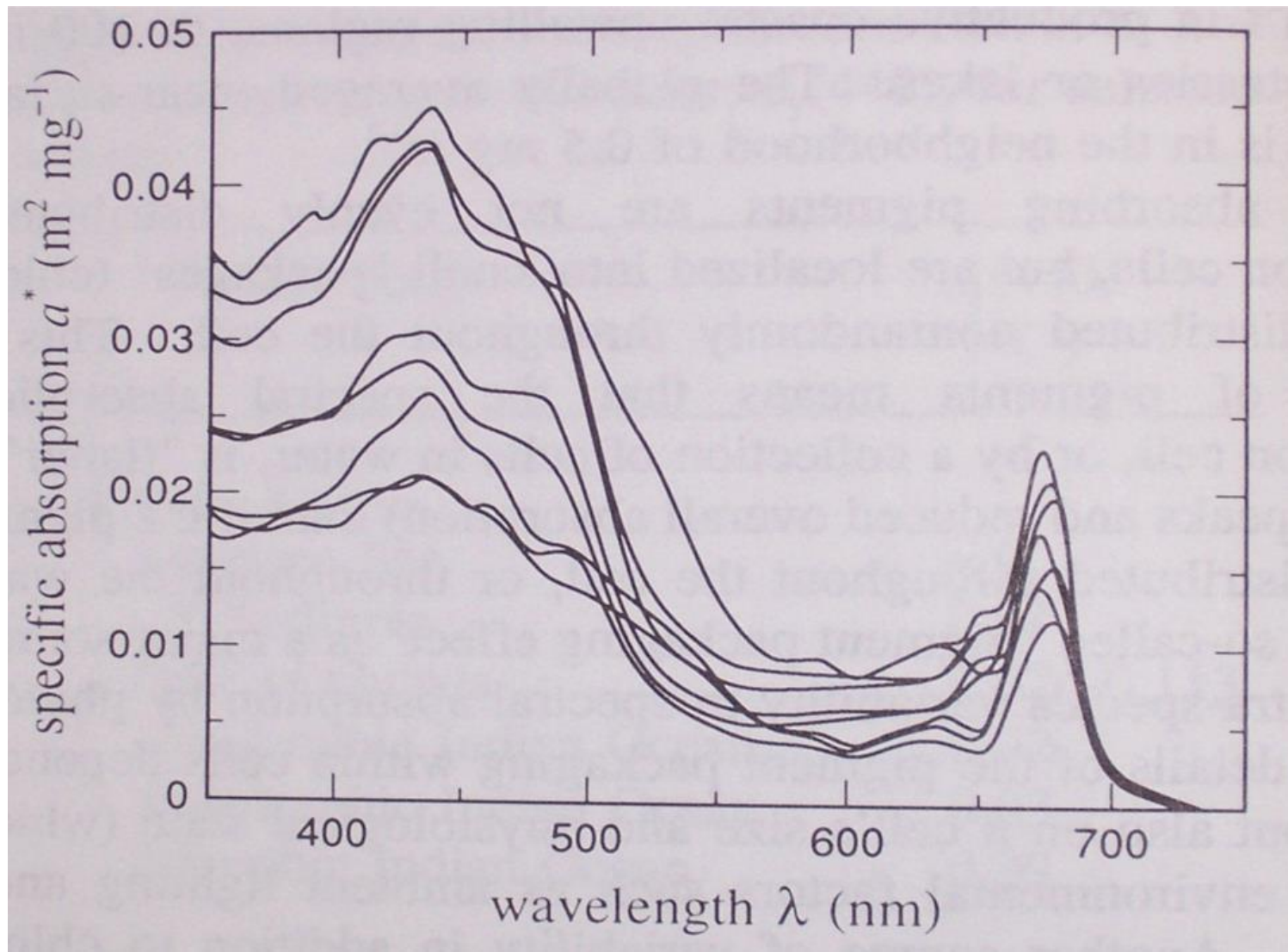
$$a(\lambda) = a_w(\lambda) + M_1 \langle a_{ph}(\lambda) \rangle + M_2 \langle a_{dg}(\lambda) \rangle$$

$$b_b(\lambda) = b_{bw}(\lambda) + b_{bp}(\lambda)$$

$$b_b(\lambda) = b_{bw}(\lambda) + M_3 \langle b_{bp}(\lambda) \rangle$$



Bio-optical models (forward model)



Modeling a_{ph} spectrum

Example of **one** parameter hyperspectral $a_{ph}(\lambda)$ model:

Bricaud et al (1995):

$$a_{ph}(\lambda) = A_{ph}(\lambda) \text{Chl}^{1-B_{ph}(\lambda)}$$

Lee (1994); Lee et al (1998):

$$a_{ph}(\lambda) = (a_0(\lambda) + a_1(\lambda) \ln(P)) P$$

$$P = a_{ph}(440)$$

Table 2. Spectral Values of the Constants Obtained When Fitting the Variations of $a_{ph}(\lambda)$ Versus the (chl $a + \text{div } a$) Concentration (Chl) to Power Laws of the Form $a_{ph}(\lambda) = A(\lambda) (\text{Chl})^{-B(\lambda)}$ and Determination Coefficients on the Log-Transformed Data r^2

λ , nm	A	B	r^2	λ , nm	A	B
400	0.0263	0.282	0.702	402	0.0271	0.281
404	0.0280	0.282	0.706	406	0.0290	0.281
408	0.0301	0.282	0.710	410	0.0313	0.283
412	0.0323	0.286	0.718	414	0.0333	0.291
416	0.0342	0.293	0.725	418	0.0349	0.296
420	0.0356	0.299	0.733	422	0.0359	0.306
424	0.0362	0.313	0.746	426	0.0369	0.316
428	0.0376	0.317	0.749	430	0.0386	0.314
432	0.0391	0.318	0.750	434	0.0395	0.324
436	0.0399	0.328	0.757	438	0.0401	0.332
440	0.0403	0.332	0.762	442	0.0398	0.339
444	0.0390	0.348	0.774	446	0.0383	0.355
448	0.0375	0.360	0.783	450	0.0371	0.359
452	0.0365	0.362	0.783	454	0.0358	0.366
456	0.0354	0.367	0.789	458	0.0351	0.368
460	0.0350	0.365	0.789	462	0.0347	0.366
464	0.0343	0.368	0.792	466	0.0339	0.369
468	0.0335	0.369	0.793	470	0.0332	0.368
472	0.0325	0.371	0.792	474	0.0318	0.375
476	0.0312	0.378	0.793	478	0.0306	0.379
480	0.0301	0.377	0.791	482	0.0296	0.377
484	0.0290	0.376	0.788	486	0.0285	0.373
488	0.0279	0.369	0.783	490	0.0274	0.361
492	0.0267	0.356	0.774	494	0.0258	0.349
496	0.0249	0.341	0.763	498	0.0240	0.332
500	0.0230	0.321	0.747	502	0.0220	0.311
504	0.0209	0.300	0.722	506	0.0199	0.288
508	0.0189	0.275	0.686	510	0.0180	0.260
512	0.0171	0.249	0.641	514	0.0163	0.237
516	0.0156	0.224	0.578	518	0.0149	0.211
520	0.0143	0.196	0.498	522	0.0137	0.184
524	0.0131	0.173	0.417	526	0.0126	0.162
528	0.0121	0.151	0.332	530	0.0117	0.139
532	0.0113	0.129	0.248	534	0.0108	0.119
536	0.0104	0.109	0.176	538	0.0100	0.100
540	0.0097	0.090	0.116	542	0.0093	0.081

Table 2. Parameters for the Empirical $a_{ph}(\lambda)$ Simulation by Eq. (12)^a

Wavelength	$a_0(\lambda)$	$a_1(\lambda)$
390	0.5813	0.0235
400	0.6843	0.0205
410	0.7782	0.0129
420	0.8637	0.006
430	0.9603	0.002
440	1.0	0
450	0.9634	0.006
460	0.9311	0.0109
470	0.8697	0.0157
480	0.789	0.0152
490	0.7558	0.0256
500	0.7333	0.0559
510	0.6911	0.0865
520	0.6327	0.0981
530	0.5681	0.0969
540	0.5046	0.09
550	0.4262	0.0781
560	0.3433	0.0659
570	0.295	0.06
580	0.2784	0.0581

Multiple-parameters model:

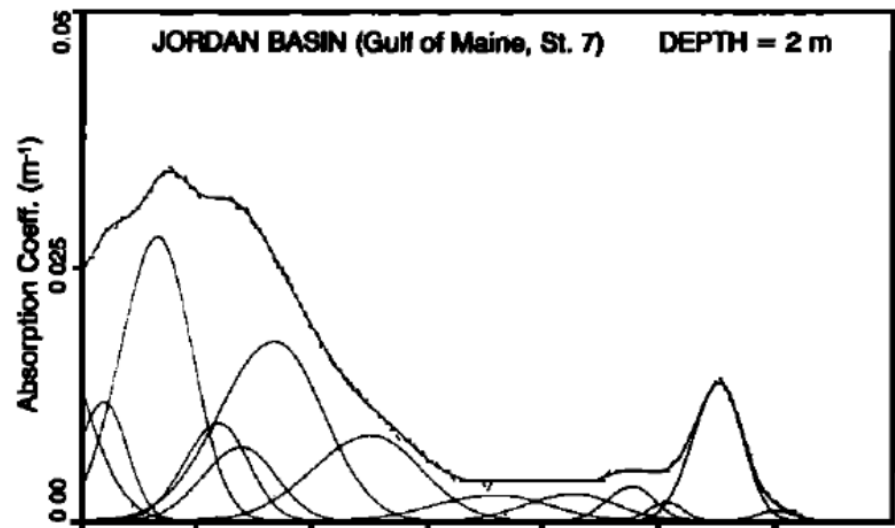
$$a_{ph}(\lambda) = \sum_{j=1}^l C_j a_j^*(\lambda_{mj}) \exp \left[\frac{(\lambda - \lambda_{mj})^2}{2\sigma_j^2} \right] \quad (7)$$

TABLE 2. Input Values and Mean Characteristics of Gaussian Bands Reflecting Absorption by Chlorophylls and Carotenoids

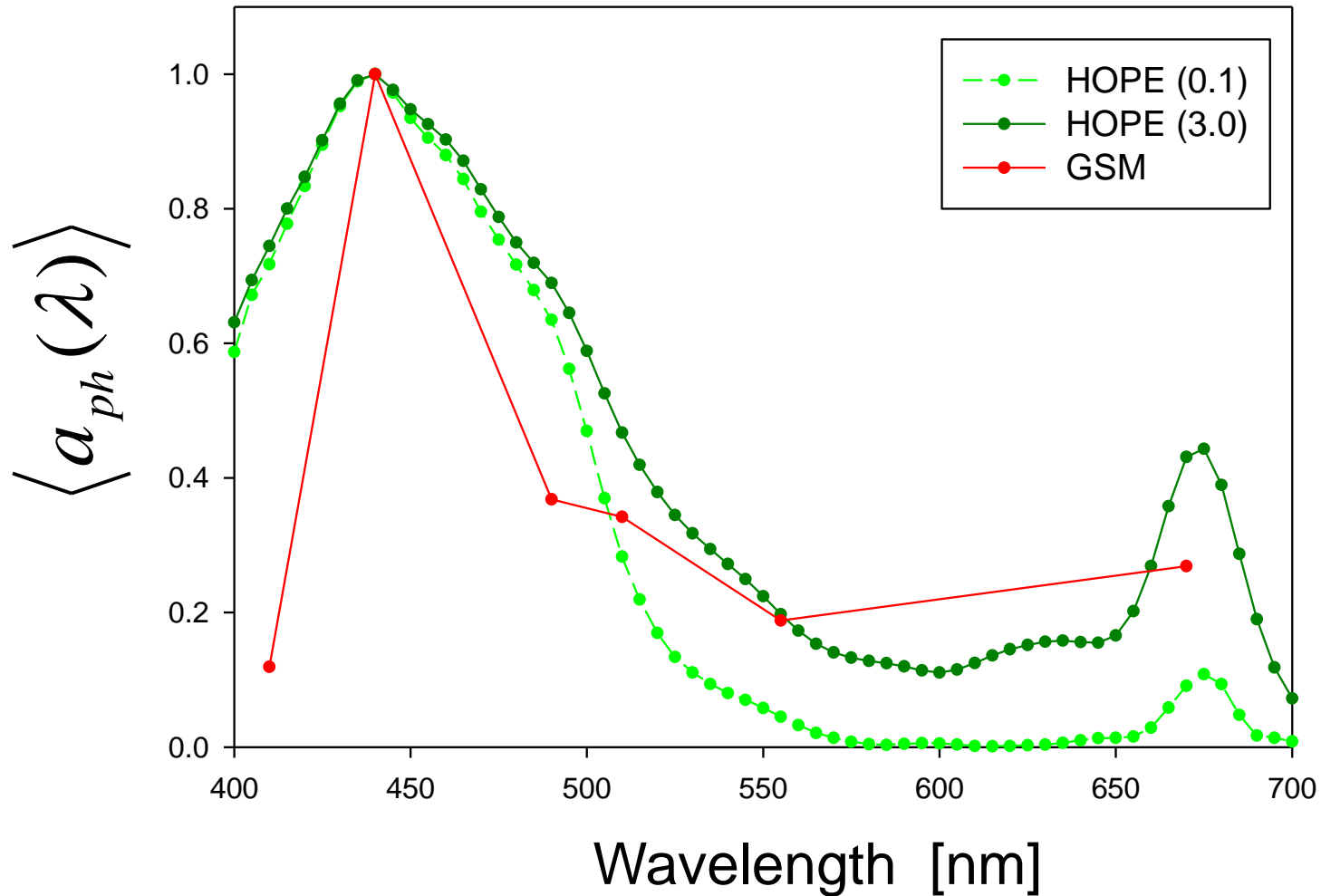
Characteristic	Gaussian Band Number and Associated Pigment Species												
	1 chl a	2 chl a	3 chl a	4 chl c	5 chl b	6 carot.	7 carot.	8 chl c	9 chl a	10 chl c	11 chl b	12 chl a	13 chl a
Input parameters													
Half width, nm	53.8	21.3	32.1	27.2	45.0	45.4	45.9	46.3	35.0	28.9	24.4	21.6	33.5
Center, nm	384	413	435	461	464	490	532	583	623	644	655	676	700
Output parameters													
Half width, nm	43.2	22.7	34.5	29.3	36.0	46.8	49.6	46.8	38.0	24.9	25.4	24.7	29.0
Center, nm	381.5	410.8	433.5	459.2	466.6	487.8	532.0	585.6	620.6	640.7	652.9	675.6	699.8
Specific absorption coefficient, m ² (mg pigment) ⁻¹	0.042	0.019	0.047	0.110	0.115	0.035	0.019	0.044	0.005	0.044	0.029	0.021	0.002

Specific absorption coefficients of each pigment are ratios of Gaussian band absorptions (in reciprocal meters) to high-performance liquid chromatography concentrations (in milligrams per cubic meter) of that pigment. Chl, chlorophyll; Carot., carotenoid.

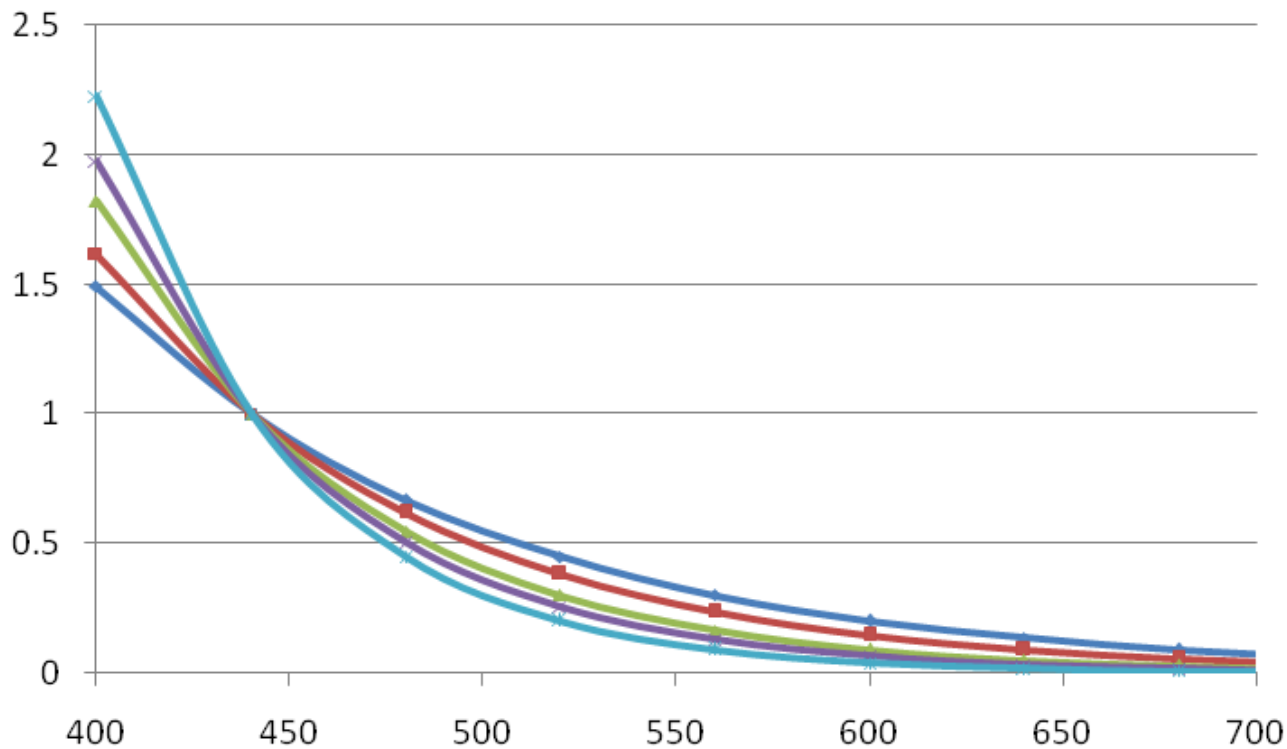
(Hoepffner and Sathyendranath, 1993)



Examples of modeled $\langle a_{ph}(\lambda) \rangle$ spectra



Absorption components: a_{dg} spectrum shapes



$$\langle a_{dg}(\lambda) \rangle = e^{-S(\lambda-440)}$$

$$S: 0.01 - 0.02 \text{ nm}^{-1}$$

(Bricaud et al 1981)

$$\langle b_{bp}(\lambda) \rangle = \left(\frac{440}{\lambda} \right)^\eta$$

$$R_{rs}(\lambda) = G \frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)}$$



$$R_{rs}(\lambda) = G \frac{b_{bw}(\lambda) + M_3 \langle b_{bp}(\lambda) \rangle}{a_w(\lambda) + M_1 \langle a_{ph}(\lambda) \rangle + M_2 \langle a_{dg}(\lambda) \rangle + b_{bw}(\lambda) + M_3 \langle b_{bp}(\lambda) \rangle}$$

3-variable model to describe an Rrs spectrum

(Sathyendranath et al 1989)

M_{1-3} are wavelength independent variables!

Then they could be derived by comparing the modeled Rrs spectrum with the measured Rrs spectrum.

Spectral ranges used for solutions (e.g. examples of BUS):

The **blue-green** domain: e.g., Hoge and Lyon (1996), Carder et al (1999)

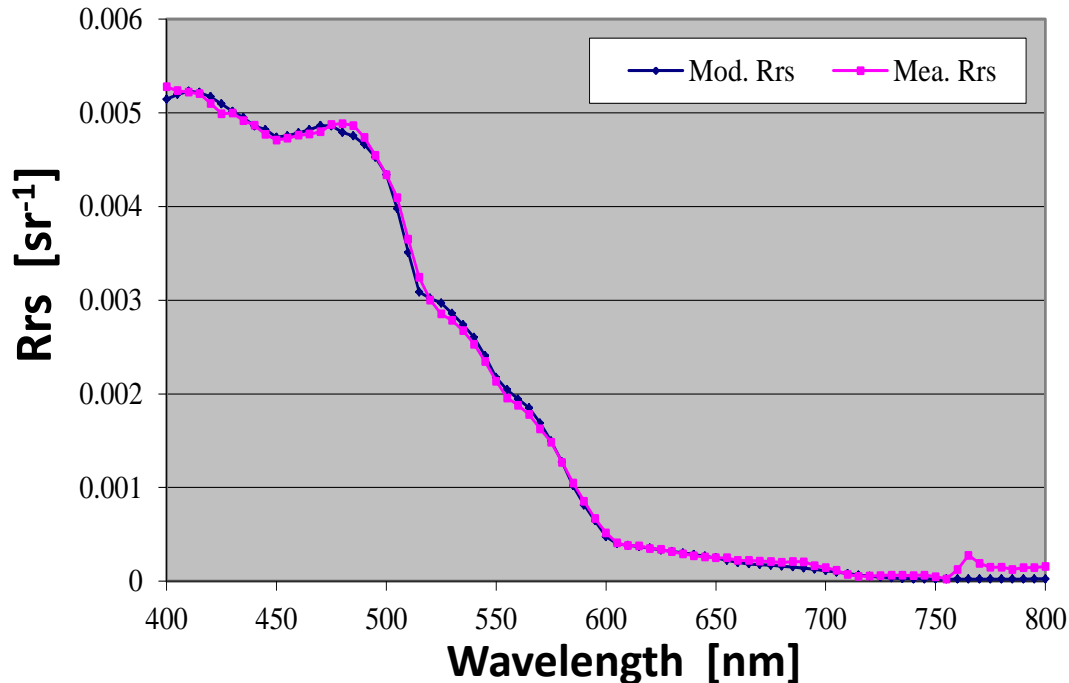
The **red-infrared** domain: e.g., Binding et al (2012)

The **entire spectrum (spectral optimization)**: e.g., Bukata et al (1995), Lee et al (1994,1996,1999), Maritorena et al (2002), Boss and Roesler (2006), Brando et al (2012), Werdell et al (2013)

Look-Up-Tables (LUT): e.g., Carder et al (1991); Mobley et al (2005)

Spectral Optimization

Matching between measured and modeled R_{rs}



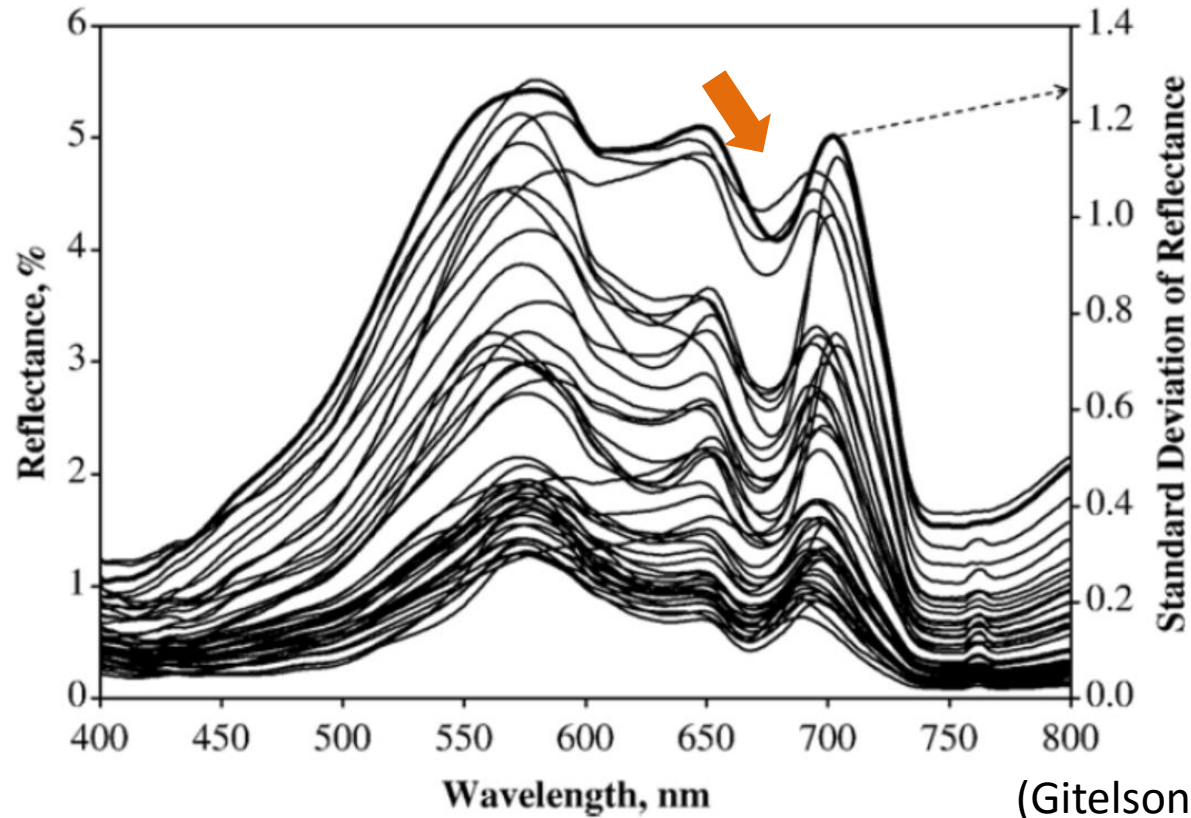
Quantitative measure of the closure (**error function**):

$$\delta_{R_{rs}} = \frac{\sqrt{\Lambda_{\lambda_1}^{\lambda_2} (\tilde{R}_{rs}(\lambda) - R_{rs}(\lambda))^2}}{\Lambda_{\lambda_1}^{\lambda_2} R_{rs}(\lambda)} = \sqrt{n} \frac{\sqrt{\sum_{\lambda_1}^{\lambda_2} (\tilde{R}_{rs}(\lambda) - R_{rs}(\lambda))^2}}{\sum_{\lambda_1}^{\lambda_2} R_{rs}(\lambda)}$$

Logic (assumption) behind SOA:

A unique set of bio-optical properties for each R_{rs} spectrum.

Algorithms using information in the red-infrared bands



Two bands

$$Chl = f\left(\frac{Rrs(75x)}{Rrs(67x)}\right)$$

Three bands

$$Chl = f\left(\frac{Rrs(75x)}{Rrs(67x)} - \frac{Rrs(75x)}{Rrs(70x)}\right)$$

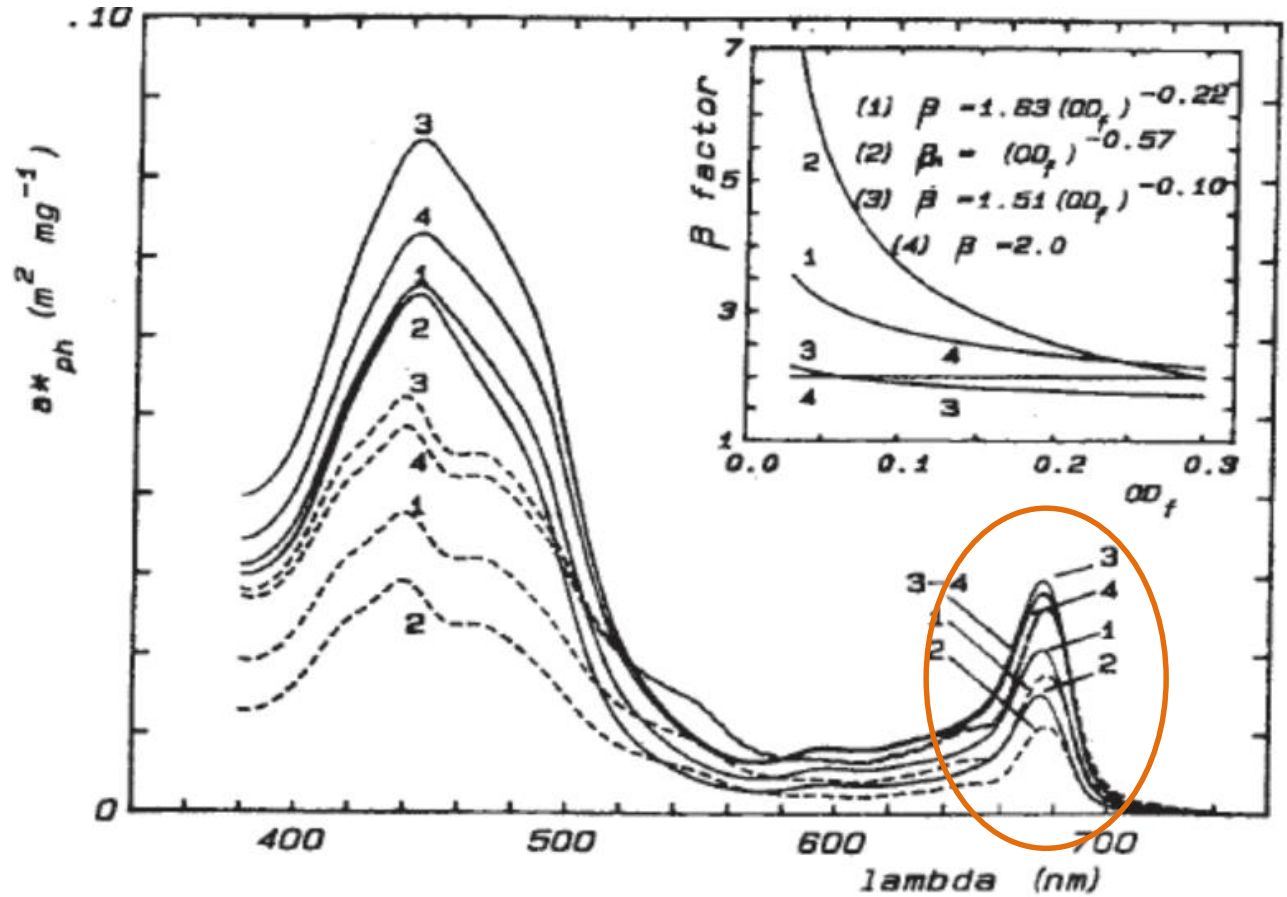
$$R_{rs}(\lambda) = G \frac{b_{bw}(\lambda) + M_3 \langle b_{bp}(\lambda) \rangle}{a_w(\lambda) + M_1 \langle a_{ph}(\lambda) \rangle + M_2 \langle a_{dg}(\lambda) \rangle + b_{bw}(\lambda) + M_3 \langle b_{bp}(\lambda) \rangle}$$

$$R_{rs}(\lambda) \approx G \frac{M_3 \langle b_{bp}(\lambda) \rangle}{a_w(\lambda) + M_1 \langle a_{ph}(\lambda) \rangle}$$

$$\frac{R_{rs}(\lambda_{red1})}{R_{rs}(\lambda_{red2})} = \frac{\langle b_{bp}(\lambda_{red1}) \rangle a_w(\lambda_{red2}) + M_1 \langle a_{ph}(\lambda_{red2}) \rangle}{\langle b_{bp}(\lambda_{red2}) \rangle a_w(\lambda_{red1}) + M_1 \langle a_{ph}(\lambda_{red1}) \rangle}$$

Proper contrast of Rrs at λ_1 and λ_2 then leads to M_1 .

a_{ph}



2. Top-down strategy (TDS):

$$R_{rs} = G \frac{b_b}{a + b_b}$$

$$R_{rs} \rightarrow b_b \& a \rightarrow a_x$$



Clarity (Secchi depth, light depth, TSM/SPM, etc)

Remote sensing measures the **total** effect:

Water clarity (or turbidity) is also a measure of total effect.

Examples of TDS:

Loisel & Stramski (2000), QAA (Lee et al, 2002); Smyth et al (2006);
Doran et al (2007).

The Quasi-Analytical Algorithm (QAA)

Forward modeling:

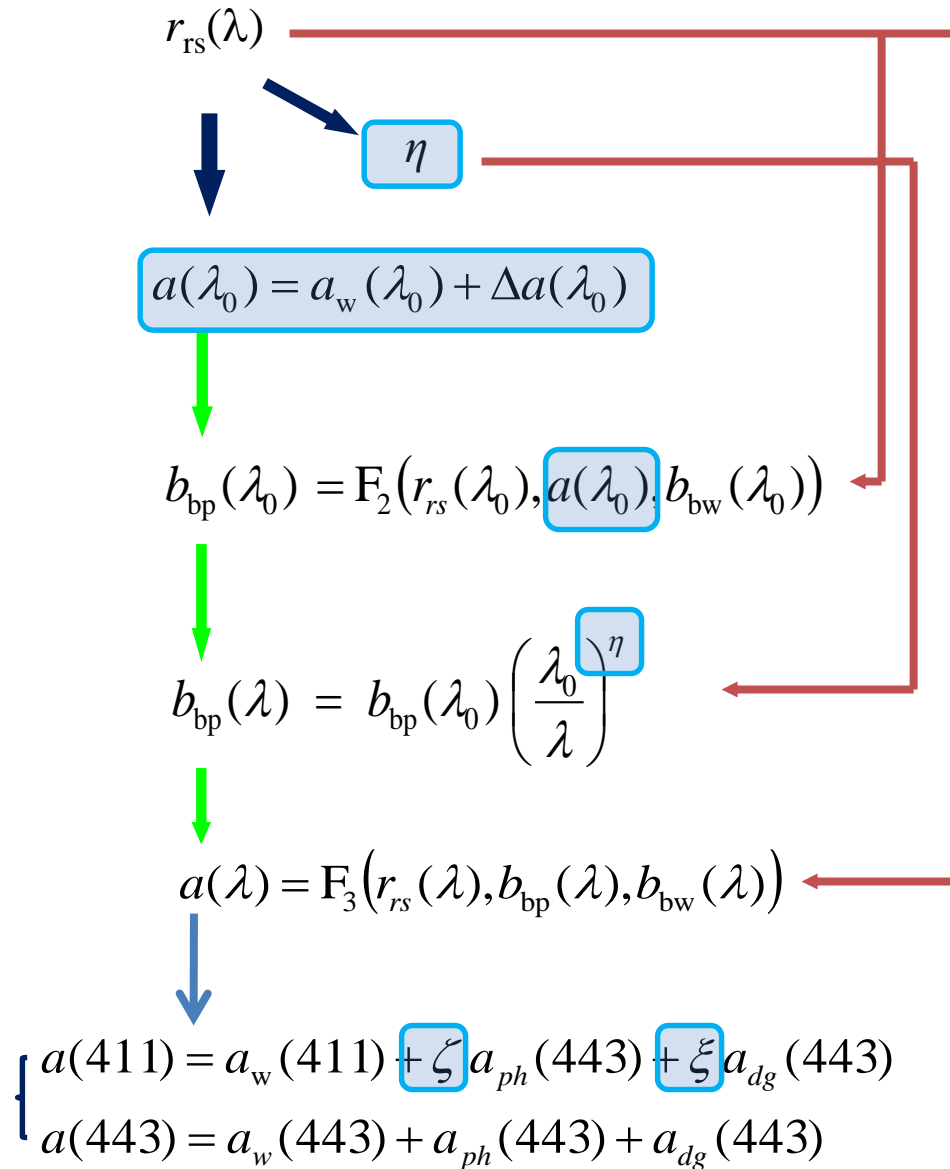
(a, b_b, etc)  Rrs

$$R_{rs} \approx 0.05 \frac{b_b}{a + b_b}$$

QAA:

(a, b_b)  Rrs

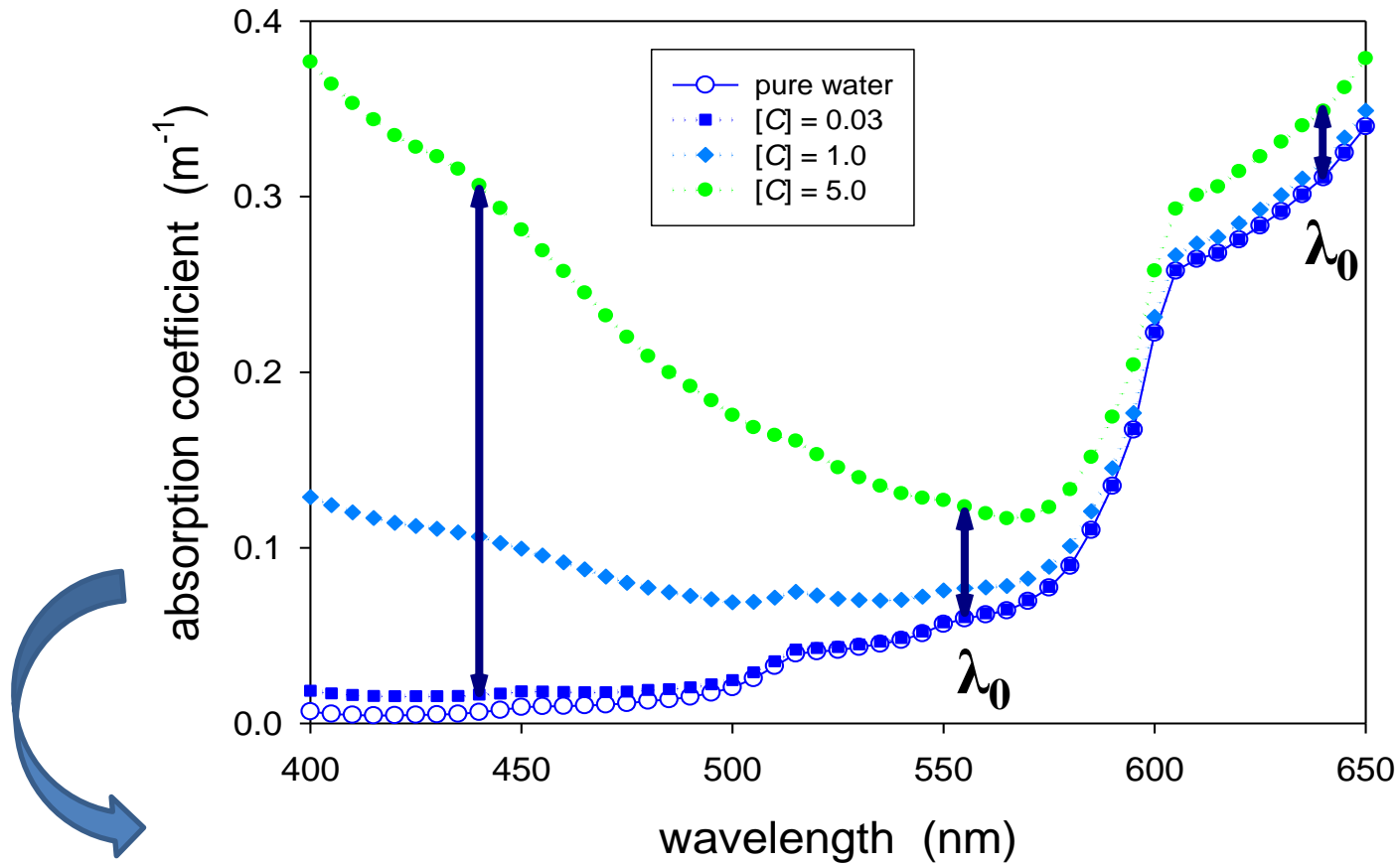
The data flow of QAA:



$$\zeta = \frac{a_{ph}(411)}{a_{ph}(443)}$$

$$\xi = \frac{a_{dg}(411)}{a_{dg}(443)}$$

Logic behind QAA (and its updated versions):



For a reference wavelength, λ_0 , variation of $a(\lambda_0)$ is limited.

Known $a(\lambda_0)$, enables calculation of $b_b(\lambda_0)$ from $R_{rs}(\lambda_0)$; propagate $b_b(\lambda_0)$ to $b_b(\lambda)$, then enables calculation of $a(\lambda)$ from $R_{rs}(\lambda)$.

No need of spectral model of $a_x(\lambda)$ in this process!

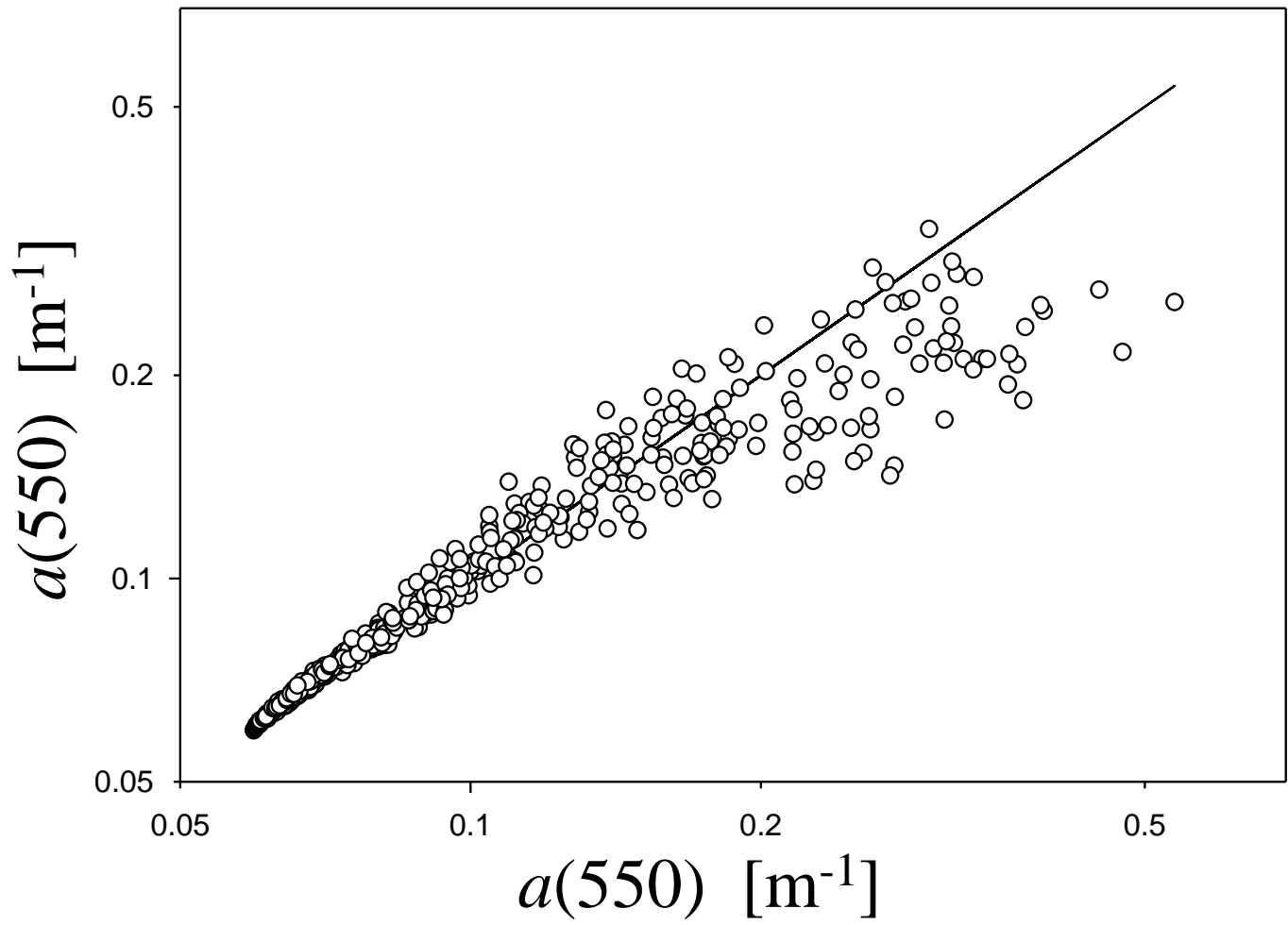
When 550 nm as the reference wavelength (λ_0)

$$a(550) = a_w(550) + 10^{-1.146 - 1.366\chi - 0.469\chi^2}$$

$$\chi = \log \left(\frac{r_{rs}(443) + r_{rs}(490)}{r_{rs}(\lambda_0) + 5 \frac{r_{rs}(667)}{r_{rs}(490)} r_{rs}(667)} \right)$$

$$a_w(550) = 0.0565$$

Empirical!



Invert Rrs:

$$R_{rs} \longrightarrow r_{rs} \longrightarrow \{a \& b_b\}$$

$$r_{rs}(\lambda) = R_{rs}(\lambda) / (0.52 + 1.7 R_{rs}(\lambda))$$

$$r_{rs} = g_0 \left(\frac{b_b}{a + b_b} \right) + g_1 \left(\frac{b_b}{a + b_b} \right)^2 \quad g_0 = 0.089, g_1 = 0.125$$

$$u(\lambda) = \frac{b_b(\lambda)}{a(\lambda) + b_b(\lambda)} = \frac{-g_0 + \sqrt{(g_0)^2 + 4 g_1 * r_{rs}(\lambda)}}{2 g_1}$$

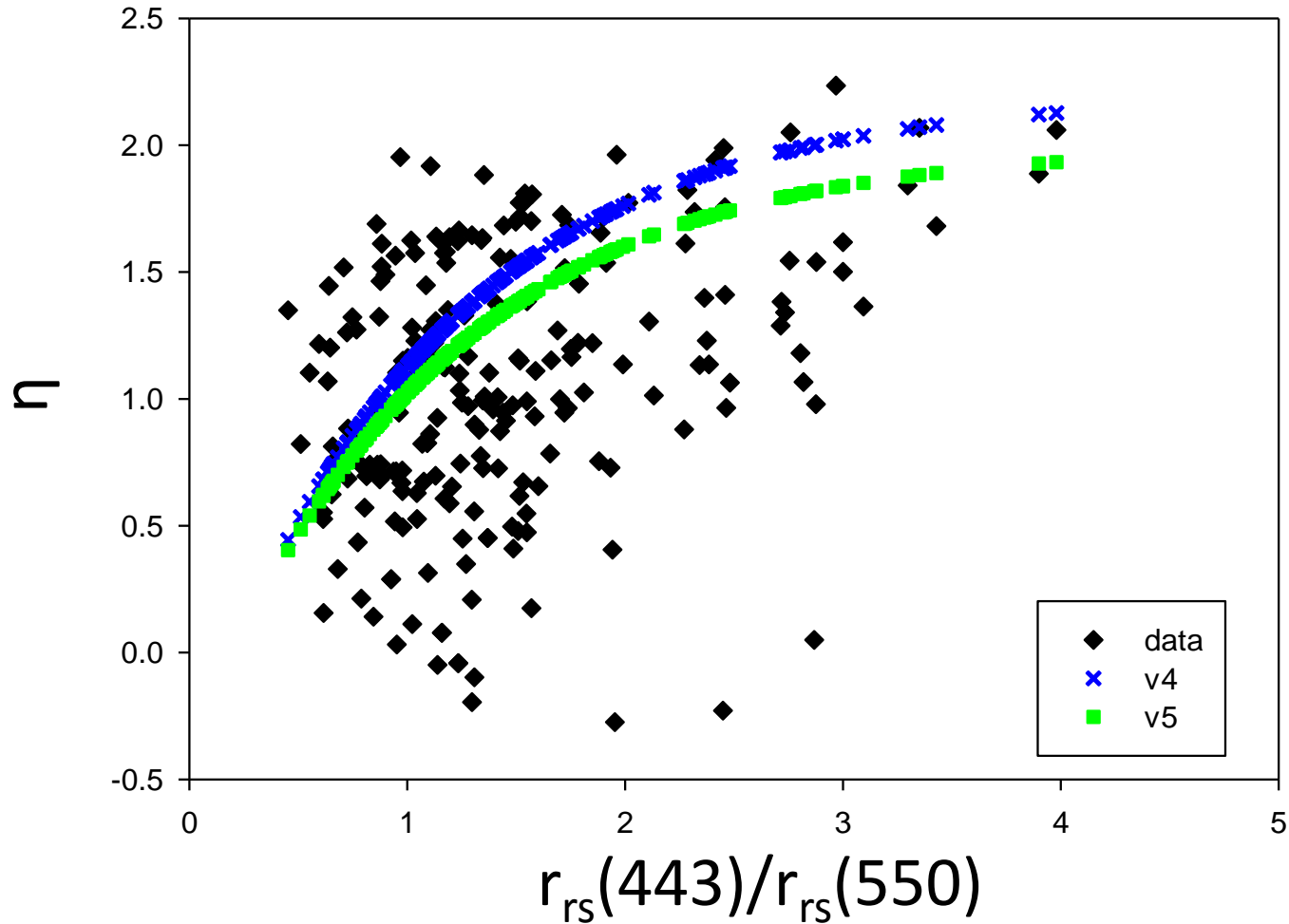
$$a(550) = a_w(550) + 10^{-1.146 - 1.366\chi - 0.469\chi^2}$$

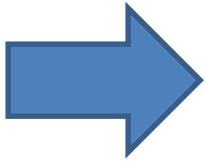
➔
$$b_{bp}(550) = \frac{u(550) a(550)}{1 - u(550)} - b_{bw}(550)$$

$$b_{bp}(\lambda) = b_{bp}(550) \left(\frac{550}{\lambda} \right)^\eta$$

Empirical:

$$\eta = 2.0 \left(1 - 1.2 \exp \left(-0.9 \frac{r_{rs}(443)}{r_{rs}(550)} \right) \right)$$





$$b_b(\lambda) = b_{bw}(\lambda) + b_{bp}(\lambda)$$



$$a(\lambda) = \frac{(1 - u(\lambda)) b_b(\lambda)}{u(\lambda)}$$

$$a(\lambda) = a_w(\lambda) + a_{ph}(\lambda) + a_{dg}(\lambda)$$



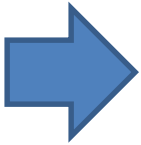
$$\begin{cases} a(410) = a_w(410) + a_{ph}(410) + a_{dg}(410), \\ a(440) = a_w(440) + a_{ph}(440) + a_{dg}(440). \end{cases}$$

$$\zeta = \frac{a_{ph}(410)}{a_{ph}(440)}$$



$$\xi = \frac{a_{dg}(410)}{a_{dg}(440)}$$

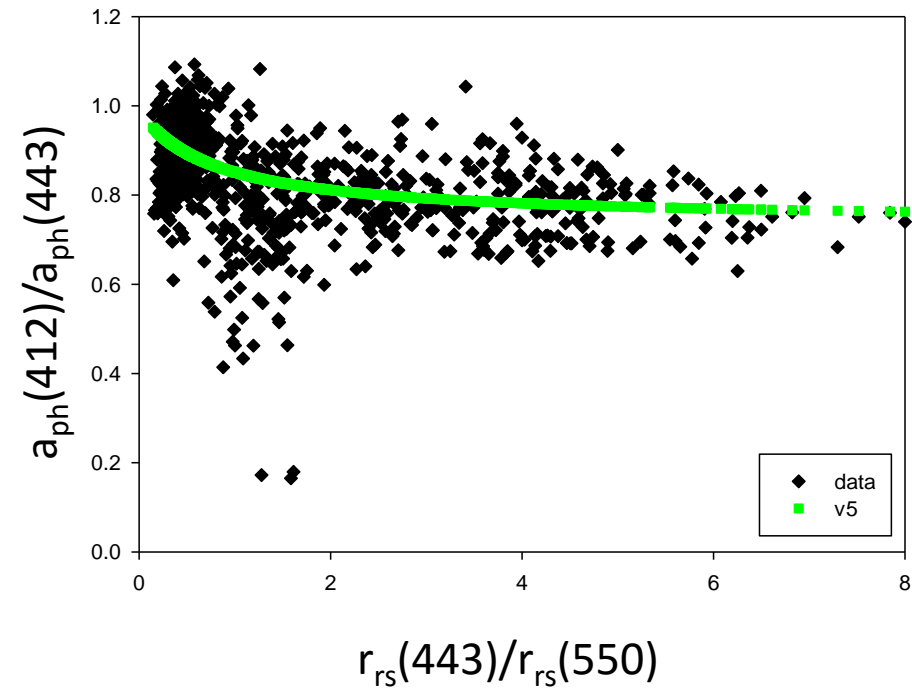
$$\begin{cases} a(410) = a_w(410) + \zeta a_{ph}(440) + \xi a_{dg}(440), \\ a(440) = a_w(440) + a_{ph}(440) + a_{dg}(440). \end{cases}$$



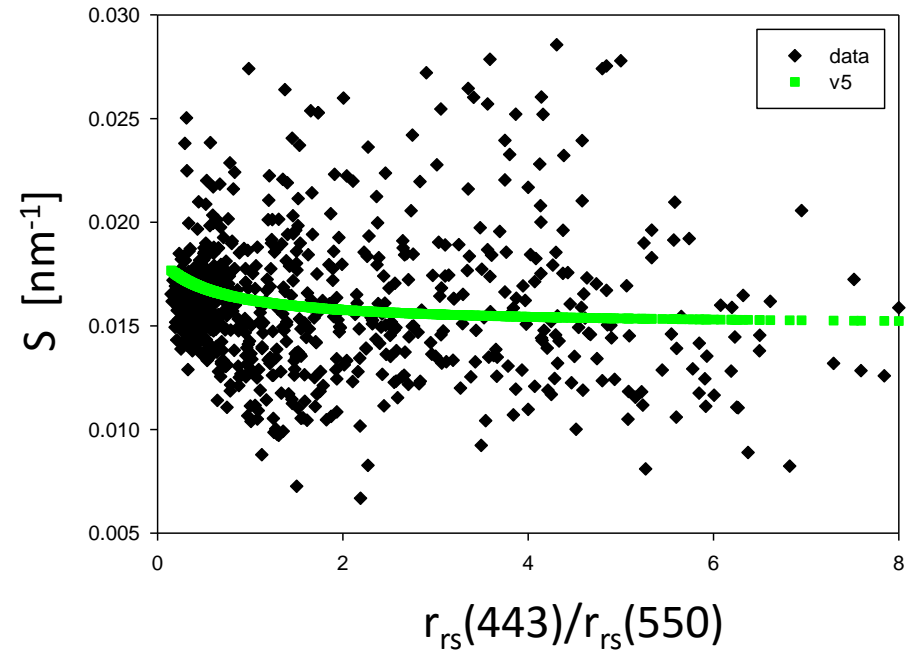
$$\begin{cases} a(410) = a_w(410) + \zeta a_{ph}(440) + \xi a_{dg}(440), \\ a(440) = a_w(440) + a_{ph}(440) + a_{dg}(440). \end{cases}$$



$$\begin{cases} a_g(440) = \frac{(a(410) - \zeta a(440)) - (a_w(410) - \zeta a_w(440))}{\xi - \zeta}, \\ a_{ph}(440) = a(440) - a_w(440) - a_{dg}(440). \end{cases}$$



$$\zeta = 0.74 + \frac{0.2}{0.8 + r_{rs}(443)/r_{rs}(550)}$$



$$\xi = e^{S(443-411)},$$

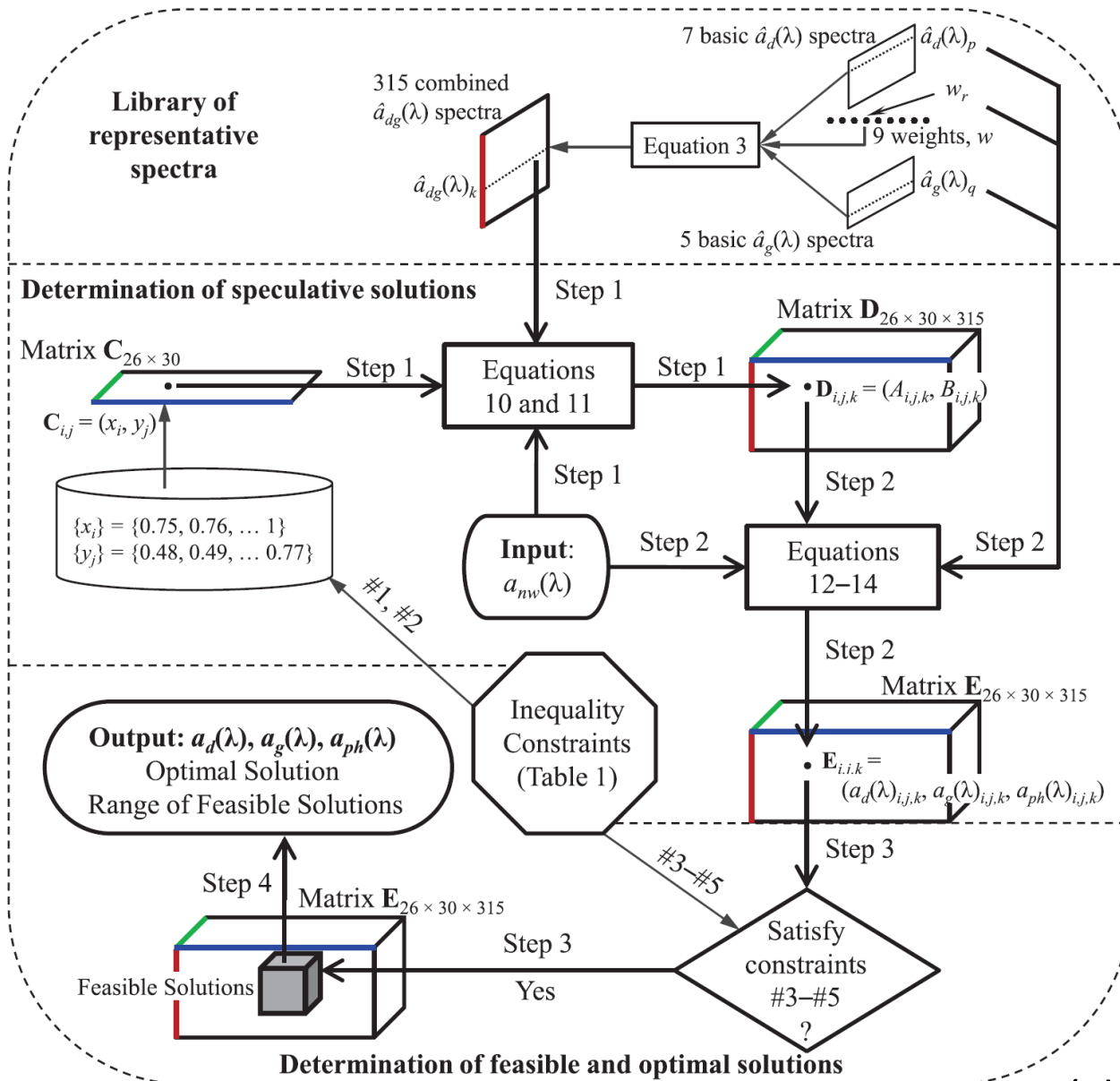
$$S = 0.015 + \frac{0.002}{0.6 + r_{rs}(443)/r_{rs}(550)}$$

Empirical!

Ensemble solutions:

Wang et al. 2006; Zheng et al. 2015

$$a(\lambda) = a_w(\lambda) + \mathbf{M}_1 \langle a_{ph}(\lambda) \rangle + \mathbf{M}_2 \langle a_{dg}(\lambda) \rangle$$



(Zheng et al. 2015)

Key Points:

1. Various inversion algorithms for IOPs have been developed; but more/better ones are also expected.
2. BUS derives every component first, then (simultaneously) derives the total optical property.

Assume the spectral shapes of the optically active components are well characterized!
BUS relies more on the accuracy of forward bio-optical model

3. TDS derives total first, then decompose to separate components.

TDS relies more on the accuracy of Rrs measurement