Course Notes for Ocean Colour Remote Sensing Course
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Module 1: Satellite Orbits

prepared
by
Assoc Professor Mervyn J Lynch
Remote Sensing and Satellite Research Group
School of Applied Science
Curtin University of Technology
PO Box U1987
Perth Western Australia 6845
AUSTRALIA
tel +618-9266-7540
fax +618-9266-2377
email <tlynchmj@cc.curtin.edu.au>

Module 1: Satellite Orbits

1.0 Artificial Earth Orbiting Satellites

The early research on orbital mechanics arose through the efforts of people such as Tyco Brahe, Copernicus, Kepler and Galileo who were clearly concerned with some of the fundamental questions about the motions of celestial objects. Their efforts led to the establishment by Keppler of the three laws of planetary motion and these, in turn, prepared the foundation for the work of Isaac Newton who formulated the Universal Law of Gravitation in 1666: namely, that

$$\mathbf{F = GmM/r^2}, \qquad \mathbf{(1)}$$

Where F = attractive force (N),
r = distance separating the two masses (m),
M = a mass (kg),
m = a second mass (kg),
G = gravitational constant.

It was in the very next year, namely 1667, that Newton raised the possibility of artificial Earth orbiting satellites. A further 300 years lapsed until 1957 when the USSR achieved the first launch into earth orbit of an artificial satellite - Sputnik - occurred.

Returning to Newton's equation (1), it would predict correctly (relativity aside) the motion of an artificial Earth satellite if the Earth was a perfect sphere of uniform density, there was no atmosphere or ocean or other external perturbing forces. However, in practice the situation is more complicated and prediction is a less precise science because not all the effects of relevance are accurately known or predictable. Some of these complicating perturbing forces arise from the non-spherical (oblate) shape of the earth, ocean tides, atmospheric drag, impact of the Sun (solar wind) on the atmosphere and the gravitational attraction of other planetary objects. This leads to the use of the term **osculating orbit** to describe the motion of the satellite. This expression and the associated **osculating orbital elements** used to define the motion will be discussed in further detail subsequently.

We could decide to describe the position of a satellite at any given time by its instantaneous position vector (3 components) and instantaneous velocity vector (3 components). This description would be quite adequate, if generated as a function of time. However, it is normal practice to compute the orbital descriptors asa function of time by executing a computer-based orbital model using inputs that are more appropriate to the precise formulation of the numerical model.

Because of the perturbing forces mentioned above, a simple formulation such as equation 1 is not adequate and we resort to a perturbation model to achieve the higher degree of prediction accuracy required. One such formulation, that conveniently has an analytical solution, is the so-called Brouwer-Lyddane model developed from the work of both Brouwer (1959) and Lyddane (1963). While we will discuss this model further, it should be noted that there are a number of other models that are available for this purpose.

Why do we require accurate orbital prediction models? The prime requirement is that in order to receive direct broadcast data from a satellite as it passes overhead, we need information on the time at which the satellite will rise over the horizon as well as the azimuth and zenith angles so we can direct a receiving antenna (a satellite dish) at that location. The path the satellite takes through the sky (the **ephemeris**), if known from the prediction model, enables the ground receiving antenna to track the satellite and therefore receive data continuously until the satellite sets at the completion of the overpass. The particular type of orbit into which a satellite is deployed is predetermined by what we require the satellite to observe. For example, if we wish a satellite to observe a particular region of the earth every day at approximately the same local time each day throughout the annual cycle, this would require a special orbit with a particular orbital plane precession. If we wished to observe events that evolve relatively rapidly in time (eg typhoons, tornadic storm cells, tropical cyclones), we would require a satellite that has the capacity to stare at a particular point on the Earth and record imagery of that region perhaps every hour or every 15 minutes. If we require observations of the whole Earth, at least once per day and every day (eg for observing synoptic weather systems, or changes in sea surface temperature), again we would require a specific orbit. However, if the requirement was for high spatial resolution (a so-called small ground footprint or small field of view (FOV)) we would need to compromise on daily coverage and adopt an orbit that images the whole of the Earth’s surface perhaps only once every 15 days.

Other application require that we know satellite orbits to extremely high precision. For example, in geodesy, global position systems (GPS) have revolutionised our ability to obtain our position on the Earth's surface. This is achieved however only if we know the position of constellation of associated satellites to very high accuracy (cm or mm accuracy). Similarly, satellite altimetry, which measures the height of the ocean surface relative to a hypothetical surface, the **geoid**, requires ideally millimeter accuracy in the knowledge of the distance from the satellite's orbit to the ocean surface. Such applications maintain the pressure to understand better the nature of the perturbing forces on orbits, to improve the accuracy of orbital models and to enhance our knowledge of how to improve ephemeris prediction with these models and supporting measurements.

2.0 Terminology and Coordinate Systems

2.1 Coordinate Systems

There are two coordinate systems that are usually encountered in remote sensing. These are **geocentric** and **geoedetic** systems. The prime difference in these systems is where the origin of the coordinate system is located. We will be using the geocentric system that has the centre of mass of the Earth C as the origin of coordinates. Next we must construct the coordinate system about this origin. The first axis is from the origin through the North Pole. The second axis is from the origin through the Earth’s equatorial plane in the direction of Aries (see below). The final axis is that which is perpendicular to the two previously defined axes.

Aries is a point in space the direction of which is determined by the intersection of the equatorial plane of the Earth (ie the plane that contains the equator) and the orbital plane defined by the Earth's annual motion around the Sun (see Figure 1.1) such that a line from the Sun's centre to the Earth along this direction points toward Aries at the time of the Vernal Equinox. This latter event occurs at a date near March 20 each year when the Sun's apparent position, as it moves from South to North in the sky, crosses the equatorial plane.

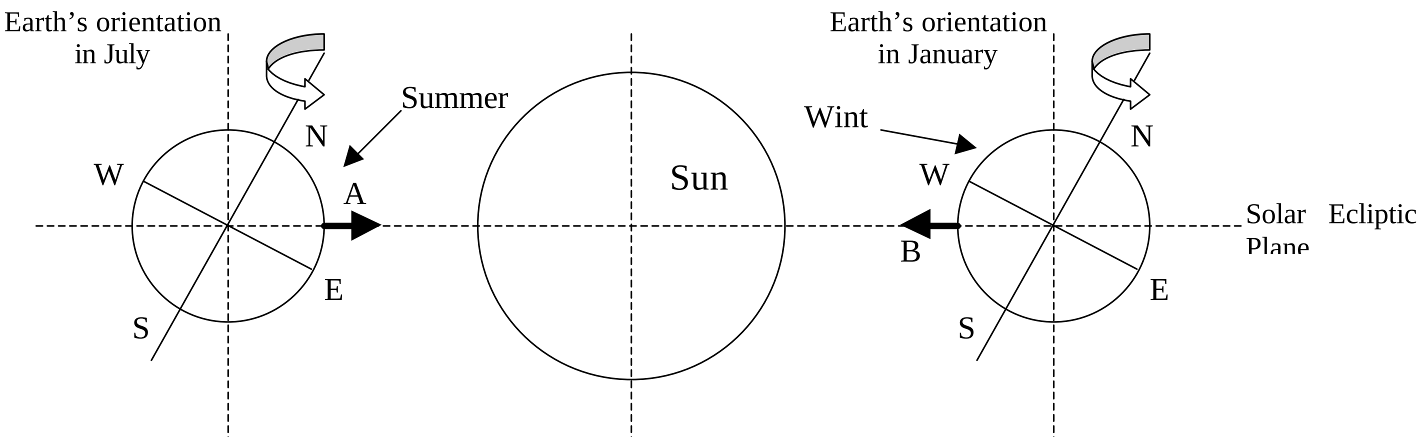

Figure 1.1 *Shown are two orientations of the Earth with respect to the Sun for the months of January and July. In July, at location A, the Sun appears directly overhead of point in the northern hemisphere. In January, at location B, the Sun appears directly overhead of this point in the southern hemisphere. Accordingly, as the Earth progresses in its orbit about the Sun in the solar ecliptic plane, it will reach a point at about March 20, when the Sun will be directly overhead of a point on the Earth's equator as the Sun makes this transition "from southern to northern latitudes". This particular time, the Vernal Equinor, defines a geometry such that a line from the Earth's centre C through the line of intersection of the solar ecliptic plane and the plane of the Earth's equator defines the direction of Aries.*

2.2 A Simple Elliptical Orbit

While orbital motion may be rather complex, we commence with the description of an elliptical orbit and define some of the important variables. With reference to figure 1.2, the orbital position r(t), at any time t, of a smaller mass m (eg a satellite) in orbit about the larger spherically symmetric and uniform density mass M, such as the Earth (note: because of spherical symmetry the larger mass may be considered a point mass with its centre of mass at C), is described by the equation of an ellipse, namely,

$$\mathbf{r = a(1 - e^2)/(1 + \cos}\ q), \quad \text{and} \qquad \textbf{(2)}$$

e = orbit **eccentricity**,
a = ellipse **semi-major axis**,
q = the **true anomaly,** measured from **perigee P** (the lowest point in the orbit where m is closest to M).

Note that **apogee A** (point in the orbit where m is furthest from M) is π radian (or 180°) removed from perigee.

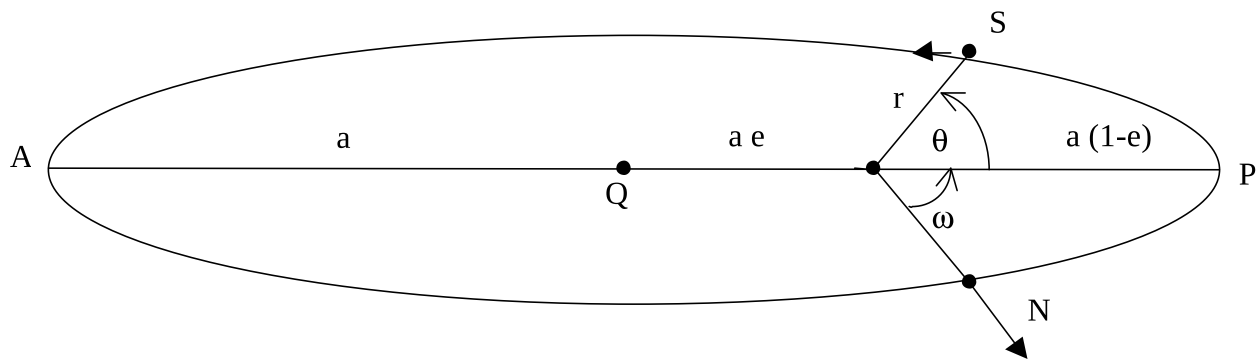

Figure 1.2 *The orbital plane of an elliptical orbit, of eccentricity e, with the satellite shown at S. the Ascending Node N, Perigee P and Apogee A are shown. The Earth's centre of mass is located at C a focus of the ellipse. The vector* $\bar{r}$ *identifies the instantaneous location of the satellite in the orbital plane. Q is the mid-point of the major axis AP of the ellipse and is of length 2a as shown by the sum of the three segments AQ, QC and CP. The angles w and q are explicitly defined in figure 3.*

The period **T** of this elliptical orbit is given by,

$$\mathbf{T = 2p(a^{3/2}/ G^{1/2}M^{1/2}),} \qquad \mathbf{(3)}$$

where, for the case of the Earth and an orbiting satellite,

G = Earth's gravitational constant, and
M = mass of the earth.

We have neglected all perturbing effects (no atmosphere so atmospheric drag is zero, no solar wind, no tidal effects, no oblateness of the Earth).

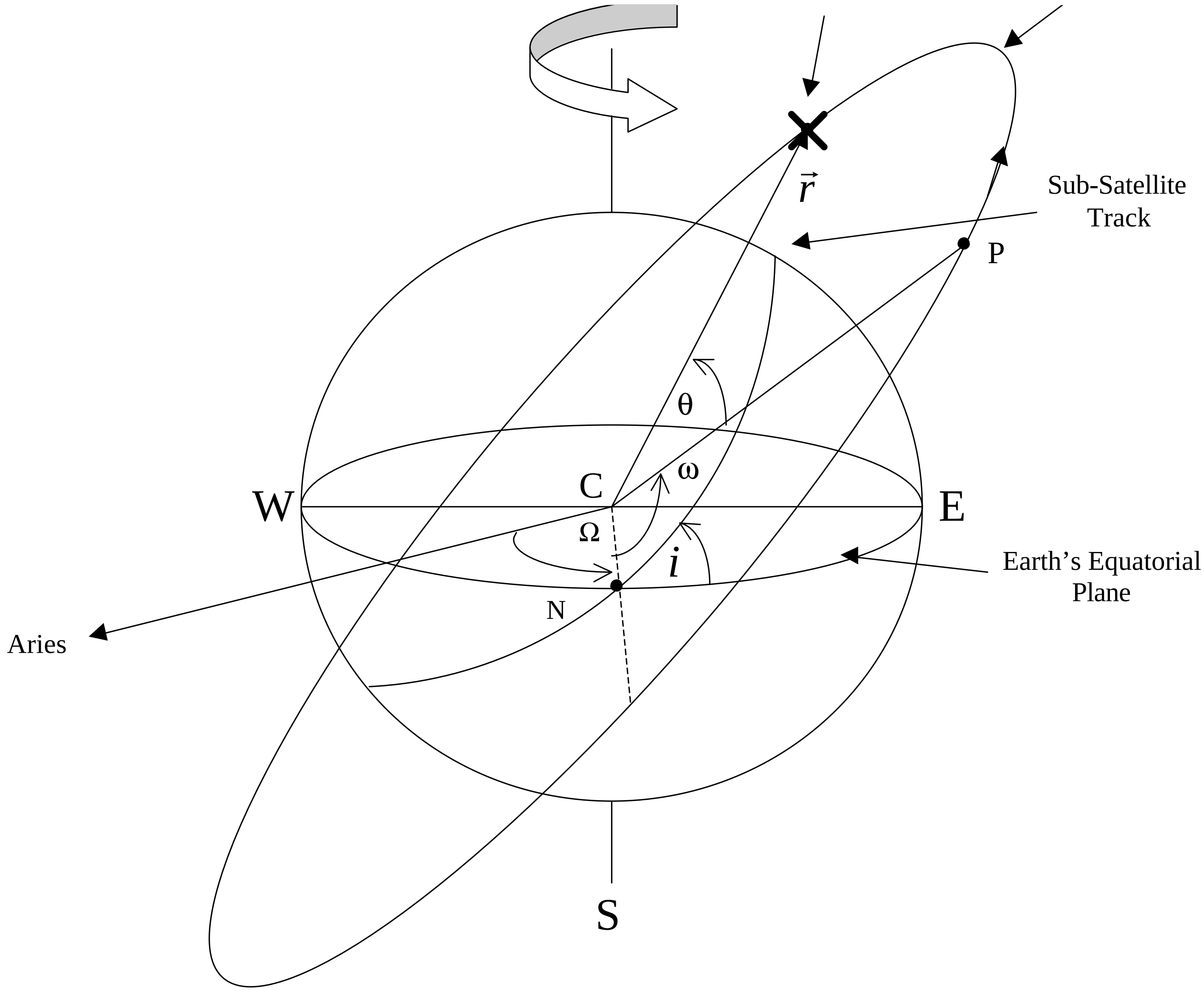

Figure 1.3: *The sub-satellite track defines an imaginary line on the Earth's surface such as would be traced out by a straight line drawn from the Earth's centre C to the instantaneous position of the satellite. When the sub-satellite track intersects the Earth's equatorial plane, as the satellite moves from south to north, it defines the location of the Ascending Node N. the orbit's inclination, i, is the angle between the plane of the satellite orbit and the Earth's equatorial plane measured anticlockwise from the latter. The Right Ascention W of the Ascending Node is the angle measured eastward from the direction of Aries to the Ascending Node N. The angle W, the Argument of Perigee, measures the angle in the satellite's orbital plane between the Ascending Node N and the Point of Perigee P. The satellite's instantaneous position in the orbital plane, the True Anomaly, is measured by the Angle q from the Point of Perigee in the sense of the direction of satellite motion to the vector* $\bar{r}$. *The term Mean Anomaly is substituted frequently for the True Anomaly.*

For the satellite orbit illustrated in Figure 1.3, the centre (of mass) of the Earth C is the origin of two of the coordinate axes. That is, an axis from C through the North Pole and a second axis directed to the Point of Aries have been defined as two coordinates. For clarity the third axis of the coordinate system has not been drawn on the diagram but it is at right angles to the two defined axes.

The satellite's **ephemeris**, namely ts position in space as a function of time, as described by the coordinate system, is shown in figure 1.3 together with the **sub-satellite track** which is that imaginary line traced out over the Earth's surface by the line joining the instantaneous position of the satellite to the Earth's centre.

The point at which the sub-satellite track crosses the equator as it crosses from the southern hemisphere to the northern hemisphere defines the location of the **Ascending Node N**. The angle measured eastward from Aries to the location of the ascending node is the Right Acsention Ω. The angle measured anti-clockwise between the equatorial plane to the sub-satellite track is the **inclination i** of the orbit. As we will see the inclination is an important satellite injection variable that controls the north-to-south latitude extent of the satellite orbit and the total number of orbits per 24 hour period.

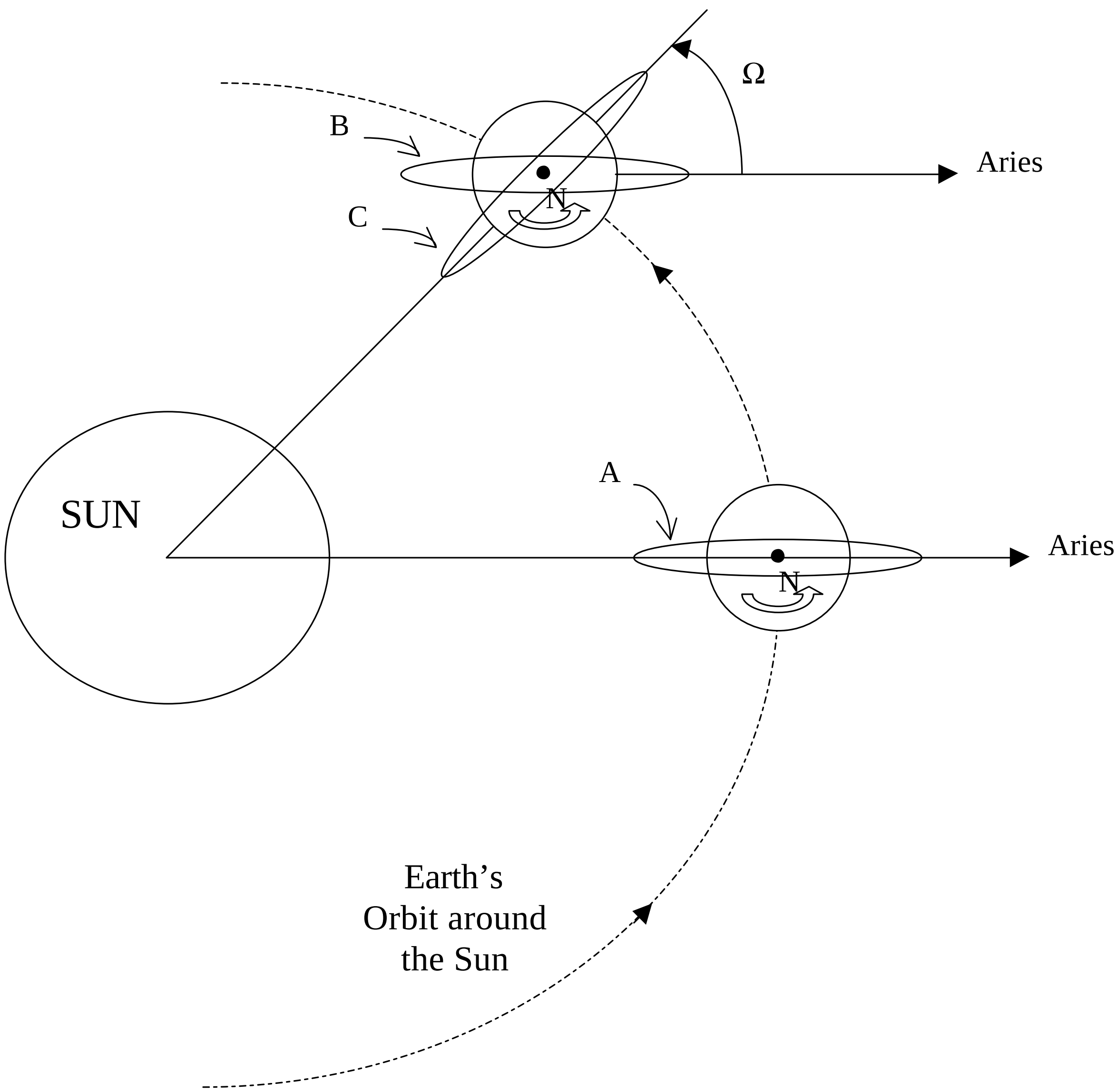

Figure 1.4 *View, from above the North Pole, two sequential positions of the Earth in its orbit around the Sun. The direction of Aries (fixed direction relative to the celestial sphere) is shown and we view the system from above the Earth's orbital plane with the location of the North Pole as shown. The satellite orbital plane A will remain fixed with respect to Aries as the Earth moves around the Sun resulting in the orbital plane B. If we wish to achieve orbit C we will need to introduce a precession of the orbital plane by changing the right Ascention W as a function of time. Thus the sequence A to B has* $\dot{\Omega}$ *= O whereas A to C requires a specific value of* $\dot{\Omega}$ *that matches the motion of the Earth around the Sun (2p radian/year). This results in a sun-synchronous orbit.*

The angle ω, the **Argument of Perigee**, is determined by the angle in the satellite's orbital plane between the Ascending Node N and the Point of Perigee P. The satellite's instantaneous position in its orbital plane, the **True Anomaly**, is described by the angle θ from the direction of the Point of Perigee (in the sense of the direction of satellite motion) to the direction of the satellite's instantaneous position vector **r.** Note that figure 1.2, which represents the satellite's orbital plane, shows these angles more clearly than figure 1.3. The term Mean Anomaly is sometimes used instead of True Anomaly.

2.3 Polar or Sun Synchronous Orbits

The value of the inclination angle i will determine key attributes of an orbit. If i =0, the orbit will be in the equatorial plane. If i > 90°, the orbit is classified as **retrograde** whereas if i < 90°, the orbit is classified **prograde**. Most Earth observing satellites are launched so as to have retrograde orbits. The reason for this is shown in figure 1.4 where the Earth is viewed from above the North Pole N. As the Earth orbits the Sun we examine the motion of the satellite's orbital plane. For the orbital configuration A, the satellite could observe a position on the Earth's surface at local noon. At configuration B, we see the satellite's orbital plane has maintained its orientation with respect to the Earth's orbital plane and the Point of Aries. The consequence of this, however, is that the satellite in orbit B now cannot make observations of the Earth at local noon. If it is important (eg for making global change; or to minimise effects due to Sun-pixel-satellite geometry – bidirectional effects) for Earth observations to be made at the same time each day over the whole annual cycle, it is clear that the Earth's orbital plane must precess; that is, move to orientation C, as shown in figure 1.4. Over an annual cycle the orbit will need to precess at the rate of 2π radian (or 360°) per year. Note that the sense of the precession is to the East and corresponds to the Right Ascention changing as a function of time. This type of orbit is term **sun-synchronous** meaning its observations are locked into a fixed (solar) time of the day. So, orbits with i > 90° are sun-syncronous and have retrograde precession of their orbits with the value of dΩ/dt positive. It is quite common for such satellite orbits to be described as **polar orbiting** satellites. Figure 1.5 illustrates the geometry for retrograde and prograde orbits.

In contrast, i < 90° classifies an orbit as prograde and the precession is to the West with Ω decreasing and dΩ/dt is negative. While it is true these orbits cross the equator and travel poleward they in general do not cross the poles. That is, an orbit of inclination angle i = 70° will only pass between the latitudes +70° and –70° and will never pass over the poles. Note, however, if i = 90°, the orbit will pass over the poles and the orbital plane will not precess.

A more detailed treatment of the orbital equations for Earth satellites is provided in Maul (1985), Stewart (1984). A particularly detailed treatment of orbital determinations for celestial and plantary objects is provided by Escobal (1976). The intention here is not to go into the details of orbital dynamics but to appreciate those aspects that are important for Earth sensing systems. Accordingly, when required the appropriate expression will be stated and typical values for earth satellite substituted to illustrate parameter values of interest.

For example, as noted above, to achieve a sun-synchronous orbit the orbital precession must be

$$\frac{d\Omega}{dt} = \frac{2\bar{u}}{365.24} rad\ \ year^{-1} \approx 1^{o}\ \ \ day^{-1} = 1\cdot 99x10^{-7}\ rad\ \ s^{-1}. \qquad (4)$$

If a satellite in a sun-synchronous circular orbit was deployed at an altitude above the Earth's surface of 1450 km, the inclination may be determined as follows.

We have,

$$\cos i = \frac{2\frac{d\Omega}{dt}a^{7/2}}{3J_2\sqrt{GM\ \ a_e}}(\ for\ eccentricity\ e = o), \qquad (5)$$

where a = radius of satellite orbit = 7828 km,
J_2=1082.63 x 10^{-6} (see below)
GM = the Earth's gravitational constant = 398, 600 $km^3 S^{-2}$
(product of G and M the gravitational constant and the mass of the Earth),
a_e = radius of the Earth = 6378 km at the equator.

This yields, $\cos i = -0\cdot 203,\ and\ \ i = 101\cdot 7^{o}.$

Hence, the orbit is retrograde and constrained to be sun-synchronous. The figures provided here are somewhat typical of the NOAA/AVHRR series of satellites of which there are normally two on orbit at any time. Normally, one is deployed with a morning ascending node and the other an afternoon ascending node.

The quantity J_2 in equation (5) is indirectly a measure on the no-sphericity of the Earth itself which is normally described as an oblate spheroid. This equatorial bulge produces a gravitational potential field (the analogue of the electrical potential with which we are more familiar). When this gravitational potential is expanded mathematically into a series of spherical harmonics, the term J_2 represents the bulk of the non-sphericity.

If we consider a circular orbit, we see that the rate of change of the True Anomaly (see Figures 1.2 and 1.3) will determine the period of the orbital motion . An approximation to d θ /dt for a circular orbit is provided by the expression,

$$\left(\frac{dq}{dt}\right)^2 \approx \frac{GM}{r^3}. \qquad (6)$$

So the period of the satellite orbit T is,

$$T = 2\overline{u}/(dq/dt) = 2\overline{u}\left[\frac{GM}{r^3}\right]^{-1/2}$$

Hence for a 1450 km altitude orbit,

$$\text{Orbital Period} = T \approx 2\overline{u}\left[\frac{(7828)^3}{398{,}600}\right]^{\frac{1}{2}} = 6892\cdot 7\ s = 114\cdot 9\ \text{min}.$$

During this interval, the Earth will have rotated 29° in an easterly direction. Hence, once this satellite has completed one orbit and returned, the point it originally observed (say, the original sub-satellite point) will now be displaced approximately 3200 km to the East (29/360 of 40,074 km, where the latter figure is the circumference of the Earth at the equator). In simple terms (neglecting orbital precession), we may think of this as the Earth rotating under a satellite orbit that maintains a fixed orientation is space. The result of this is that satellites in sun-synchronous orbits may achieve global coverage typically in one day. The projection of ephemeris onto the Earth's surface, the sub-satellite track, illustrates this very clearly.

2.4 Geostationary or Geosynchronous Orbits

An important class of satellite orbits is the geostationary or geosynchronous orbit. For such orbits the inclination angle i = 0° and the orbit lies in the equatorial plane. The normal requirement for such orbits is to place them directly over the equator at a particular longitude. For example, the GOES East and GOES West satellites are placed in orbits over the Americas (normally at 75° W and 135° W, the Japanese Geostationary Meteorological Satellite (GMS) is located over New Guinea and Meteosat over Europe. For this to occur it is clear that the orbital period of these satellites must equal the period of rotation of the Earth. That is, the satellite period is one **sidereal day** or 86,164 s.

If geostationary orbits are such that $i \neq 0°$, but is small, then the sub-satellite track is what is frequently termed a "figure 8". This type of orbit is shown in figure 1.6 for the case where i = 10°, and we see the satellite N-S extent matches the inclination angle. A non-zero i may not always be intentional. If a satellite is injected into orbit it is sometimes difficult to set i precisely to zero and, of course, valuable on-board fuel is consumed in correcting the orbit. So, a small offset is not particularly problematic as long as the "wandering" of the satellite in its "figure 8" does not stray outside of the reception angle of the fixed-direction ground station antenna.

Equation (6),with appropriate substitutions, becomes,

$$T\ (\text{min}) = 1.66\ [6378 + h\ (\text{km})]^{3/2}.10^{-4},$$

where again, h is the altitude of the satellite above the Earth's surface.

So for a geostationary satellite orbit with a period of 2π day^{-1}, we may determine h. For the values given, we see that for a geostationary orbit

$$h = [24*60 \text{ min}]/[1.66*10^{-4}] - 6378 = 35{,}843 \text{ km}.$$

Accordingly, geostationary satellites are positioned some 35 times the altitude of the typical polar orbiting satellites. A major consequence of this contrast in orbital deployment is that the geostationary satellites have to be equipped with much larger aperture (diameter) optical systems if they are to resolve spatially comparable detail on the surface of the Earth.

4.0 Computer-based Modelling of Satellite Orbits

Models of satellite orbits are routinely used to support satellite data collection from receiving stations. Typically, data downlink station or direct readout station must have predictions of when the satelliote of interst will rise over the horizon and the appropriate azimuth angle at which the receiving antenna or dish must be pointed. In order to track the satellite as it moves overhead the antenna may require a sequence of pairs of azimuth and zenith angles as a funtion of time to maintain accurate tracking. Because many of the perturbing forces (atmospheric drag, solar wind, approximations to the Earth's gravitational field as expressed by the higher order terms in the gravitational potential) are not captured in the prediction models the model predictions do not maintain their accuracy for an extended period. Typically, one runs the prediction model approximately every few days (maybe weekly - depending on the satellite) to update the orbital prediction. To achieve this with acceptable accuracy, models resort to use of **Orbital Elements** to reset the initial conditions in the computer-based model. These orbital elements are derived from direct measurement of the satellites position made using **satellite laser ranging (SLR)** instruments. These instruments locate the satellite position in the sky by reflecting a laser pulse off reflectors (corner cube reflectors) on the satellite. The time for the round trip of the laser pulse provides an accurate range (order of a cm accuracy is typical) and the azimuth and elevation at which the laser fired record the direction. By tracking the satellite across the sky the SLR defines the satellite ephemeris (ie its orbit in space and time). From these measurements a set of orbital elements are derived that are appropriate to input to the orbital prediction model. There are two common sets of orbital elements used for satellite prediction, namely T-bus elements and (NORAD) two-line elements. Several of these orbital elements will be familiar. here we will discuss just the two-line elements (2LE). The elements as illustrated below are provided as two-lines of consecutive characters. In order to aid location of particular elements, the special characters are used above the first line of elements and immediately below the second line of elements. The identity of those elements used in orbital prediction models will be described below.

```
  *    *   ##       ++$              $
1 21263U 91 32  B 93231.76315203   .00000177  00000-0    88271- 4  0    6491
2 21263    98.6545 260.6933 0013797  33.2603   326.9449 14.22300920117899
           !        ! ?        ? "        "  :       :   %      % /          / \      \
```

Satellite Number = 21263 (see line 1 characters between and below the asterisks (*)
Each satellite is assigned a distinct identification number.

Launch Year = 91 (line 1 below the symbols ##)
The year in which the spacecraft was launched (eg 1991)

Epoch Year = 93 (line 1, between and below (++) characters)
The year the elements were obtained (eg 1993)

Epoch Day = 231.763115203 (line 1 between and below the $ characters)
The epoch day of year in GMT or UTC (1≤ epoch day ≥ 367)

Inclination = 98.6545 (line 2 above and between the ! characters)
The angle at which the orbit plabecrosses the equatorial plane as described earlier.

Right Ascention of the Ascending Node = 260.6933 degree (line 2 between and above the ? characters). This parameter was defined earlier.

Orbit Eccentricity = 0.0013797 (line 2 between and above the " characters)
Note the decimal point is not provided in the elements. A zero to the left of the decimal point has been added post extraction to emphasise the eccentricity e is less than unity) As defined earlier the eccentricity defines the shape of the elliptical orbit. Ab eccentricity of defines a circular orbit. The degree of ellipticity increases as the eccentricity increases toward unity ($0 \leq e \leq 1$).

Argument of Perigree = 33.2603 degree (line 2 between and above the **:** characters)
Defined previously as the angle in the orbital plane from the ascending node to the point of perigee in the direction of the satellite's motion.

Mean Anomaly = 326.9449 (line 2 between and above the % characters)
The angle in the orbital plane from the the point of perigee to the line of the vector from the Earth to the satellite's position in the direction of the satellite's motion.

Mean Motion = 14.22300920 (line 2 between and above the / characters)
The mean number of orbits per day.

Orbit number = 11789 (line 2 between and above the \ characters)
The current number of Earth revolutions completed by the satellite since launch. The value is incremented when the satellite crosses the equator on an ascending pass (northward bound).

If a model requires any other elements (eg semi-major axis) they may be computed from the existing elements

A software package SeaTrack (Lambert et al, 1993) describes a particular orbital prediction model. The above description of the two-line elements has been drawn from that publication.

In the Workshop, a particular orbital prediction model will be studied in the Laboratory program. Experince will be gained in model execution and its application to the orbital prediction of the SeaStar platform on which SeaWiFS is deployed. The model selected will also permit the determination of the area over which a particular satellite overpass will deliver observational data when the satellite is tracked by a specific ground receiving station (at some predetermined lat and long). Such information is particularly useful when planning research cruises, enabling a cruise vessel to position itself favourable so that is within the geographical scene (reception area) captured by the satellite orbit for that particular day.

4.2 Model Outputs

Another orbital prediction model is available at the URL <http://eospso.gsfc.nasa.gov> and may be executed online. The type of information provided by this site is appropriate for tracking particular satellites since it provides the coordinates for initialising the antenna when the satellite rises over the horizon and permits programming the antenna tracking system's pointing direction and rate of progress as it follows the satellite's motion across the sky.

6.0 Concluding Remarks

The intention of this segment of the Workshop was to make the point that satellites are instruments, just like laboratory instruments, and we need to understand how to use them and why we use them in different ways. Further, it is apparent that we have considerable control over the precise way we deploy a satellite on-orbit so that it can make the observations appropriate to our needs. The variables used to describe a satellite's orbit are not the simple variables that we might have hoped for - for example position (x, y and z) velocity (v_x, v_y and v_z) at time t. The variables,in most cases, have their origins in astronomy. Direct read-out (or downlink) satellite stations need information so they are ready to receive satellite data transmissions as soon as the satellite appears over the horizon. This requirement has stimulated the development of models of satellite orbits. The initial conditions are provided to the models via a set of orbital elements that are in turn derived for direct observations of the satellite ephemeris using laser tracking and ranging instruments

7.0 References

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Module 2: Basic Radiometric Quantities and Definitions

prepared
by
Assoc Professor Mervyn J Lynch
Remote Sensing and Satellite Research Group
School of Applied Science
Curtin University of Technology
PO Box U1987
Perth Western Australia 6845
AUSTRALIA
tel +618-9266-7540
fax +618-9266-2377
email <tlynchmj@cc.curtin.edu.au>

Module 2: Basic Radiometric Quantities and Definitions

1.0 Geometry, Angles and Solid Angles

An angle is defined in two dimensional space as a ratio of two lengths - an arc of a circle divided by the radius of that circle - see figure 2.1. It is by definition dimensionless and the unit of angular measure is termed the **radian** (abbreviated to **rad**). When the length of the arc is equal to the radius the angle is 1 rad. If we increase the angle progressively unit we occupy a complete circle the angle now is measured by the circumference ($2\pi r$) divided by the radius r of the circle. Thus a complete circle corresponds to an angle of 2π rad or approximately 6.28 rad. Frequently angles are measured in degrees with
$360° = 2\pi$ rad.

(insert figure 2.1 and caption here)

Figure 2.1 *Definition of an angle q in two dimensions is that angle subtended by an arc of length* ***a*** *at a radius* ***r****.*

Note angle θ is a ratio of two lengths and therefore is dimensionless. If a = r, the angle *q* is 1 radian.

In the real world we more frequently encounter what are termed solid angles. For example, if we view a "circular" object (the moon) then the eye and the moon's area (its area appears to be a flat disk of πr^2 m^2 where r is now the radius of the moon) define a conical volume in space. This "angle" is termed a **solid angle** and is measured by the area of the object divided by the radius squared (in this case the eye - moon distance. The unit of solid angle measure is the **steradian** (abbreviated sterad). If now we imagine travelling toward the moon we will reduce the distance (eye - moon) but the area presented by the moon is unchanged. This causes the solid angle to increase. If now we imagine ourselves at the centre of a sphere of radius r then the surface of the sphere $4\pi r^2$ is now the appropriate "area" and it is at radius r. This is the largest solid angle we can create and it corresponds to $4\pi r^2/ r^2$ or 4π steradian. Figure 2.2 illustrates the measurement of a solid angle and introduces the azimuth and zenith angles φ and θ respectively.

(insert figure 2.2 and caption here)

Figure 2.2. *The element of solid angle dW is defined in terms of the elemental area dA at distance r from the origin O. dA is produced by small increments dq and df in the zenith angle q and azimuthal angle f respectively, so that* $d\Omega = dA/r^2$.

Note that, in figure 2.2, the elemental surface area dA has sides of length rsinθdϕ and rdθ giving the area dA as,

$$\mathbf{dA = r^2 sinq dq df} . \qquad (1)$$

2.0 The Basic Radiometric Quantities

2.1.1 Irradiance

An optical beam may be thought of as a collection or ensemble of photons each carrying energy. If this beam is incident on a surface then the rate that energy (joule or J) arrives at that surface is termed **power** may be measured in **joule s^{-1}** or **J s^{-1}** This latter unit is the definition of the **watt or W**. Frequently in radiometry the power in an optical beam per unit area (see figure 2.3a) is termed the **radiant flux d**f but more commonly the **irradiance E (**the unit is **W m^{-2}).** Hence, in figure 2.3a, the irradiance E incident on the surface due to the radiant flux dϕ falling on the area dA is dϕ/dA. This quantity will be encountered frequently in remote sensing.

In figure 2.3a the beam was normal to the surface. Figure 2.3b shows the direction of the beam now impacting the surface at an angle of incidence θ. Since the beam, of the same power, is now distributed over a larger surface area, when compared with the case in figure 2.3a, the irradiance will be reduced by the factor cos θ since the area impacted by the beam has increased from dA to dA/cosθ.

(insert figure 2.3 and caption here)

Figure 2.3. *Consider a beam of radiation of radiant flux or power dF (W) incident on a horizontal surface. In case (a) the radiant flux per unit area, the irradiance*

$$E_a = d\Phi / dA \qquad \textbf{\textit{(Wm}}^{-2)}\textbf{.}$$

For case (b) where the beam is now inclined to the surface at a zenith angle we see the area over which the beam irradiates the surface is now dA/cosq. The reduced irradiance in case (b) becomes $E_b = d\Phi / (dA / \cos q) = d\Phi \cos q / dA$.

2.1.2 Radiance

Consider the geometry illustrated in figure 2.4 where the the normal to a small element of surface dA is inclined at an angle θ to the direction from which we observe the radiant flux from the surface. The determine the radiant flux or radiant power dϕ leaving area dA (eg radiation from the ocean to the atmosphere) that fills or passes through the solid angle dΩ. This quantity is defined as the **radiance L**, where, from its definition,

$$\mathbf{L = df \ / \ (dA.cosq.dW),} \qquad (2)$$

and the **units of radiance are W m^{-2}.ster^{-1}**. Sometimes radiance exiting a surface is termed **exitance**.

(insert figure 2.4 and caption here)

Figure 2.4(a). *The radiant power dF leaving an elemental projected area dA on an extended source (such as the ocean surface) per unit solid angle dW is defined as the radiance L and expressed as,*

$$L = \frac{d\Phi}{dA \cos q d\Omega} (Wm^{-2} sr^{-1}).$$

Figure 2.4(b) *The projected area concerned, namely, dAcosq is shown.*

The quantity radiance corresponds to the type of observation we might conceive of making in practice. For example, if we viewed the small element of surface area dA in figure 2.4 from, say, an aircraft, using a telescope that had an aperture such that the solid angle involved was dΩ, then, according to equation 2, we would determine the radiance. The same circumstance applied when we make observations from a satellite platform.

It is worth noting that solid angle is important generally in characterising laboratory sources. For example, we could compare a beam from, say, a 60 W laser to that from a 60 W light bulb. The important issue here is the relative solid angles over which the radiation from these two sources is distributed. For example, these might be 10^{-5} sterad for the laser versus 4π sterad for the light bulb - a ratio of some 10^{+6} in solid angle. So it is the appropriate solid angle that is the key variable in determining the radiance from these two sources that have identical output powers. Useful sources to consult are Maul (1985), Stewart (1985) and Kirk (1986).

3.0 More About Radiance

Making measurements in radiance units is an important concept in radiometry. We define an elementary beam using figure 2.5 as an aid. The notion of an elementary beam is that all rays that pass through the aperture dA_1 also pass through aperture dA_2. If the latter holds, then the power passing through dA_1 equals the power passing through dA_2 . These two aperatures, with surface normals as shown, are centered at X and Y respectively and subtend solid angles to each other as shown in the figure. Note that θ_1 and θ_2 respectively are the angles between the surface normals and the direction of propagation of the beam of radiation, namely from X to Y.

(insert figure 2.5 and caption here)

Figure 2.5. The concept of an elementary beam is illustrated: all rays that pass through dW, and dW_2 dA, also pass through dA_2. The respective solid angles are shown along with the surface normals $\overline{U}_{n1}$ *and* $\overline{U}_{n2}$..

We now can write down the appropriate radiances at the two apertures,

$$L_1 = df_1 / (dA_1 .\ \cos q_1 .\ dW_1), \text{ and} \quad (3)$$

$$L_2 = df_2 / (dA_2 .\ \cos q_2 .\ dW_2). \quad (4)$$

However, since,

$$df_1 = df_2,$$

we have from (3) that,

$$L_1 . dA_1 . \cos q_1 . dW_1 = L_2 . dA_2 . \cos q_2 .\ dW_2. \quad (5)$$

However, using the definition of solid angle we also have that,

$$dW_1 = (dA_2 . \cos q_2)/r^2 \quad \text{and} \quad dW_2 = (dA_1 . \cos q_1)/r^2. \quad (6)$$

If we now substitute equations (6) into (5) we obtain the particularly important relationship that, for an elementary beam,

$$L_1 = L_2 \quad (7)$$

4.0 Making Earth Observations from Space

Figure 2.6 illustrates schematically how we might use a simplified sensor on a satellite to observe the surface of the Earth. Note that while we have not included lenses or mirrors in the sensor, these do not make the arguments any more complex if we observe the elementary beam concept. For example, if a ray enters the sensor foreoptics (front aperture) it is normal to ensure that that ray finally falls on the detector surface. In other words, to achieve maximum performance out of a satellite instrument, we normally design it to conform to an elementary beam system since, to do otherwise would degrade the sensor.

We have that the radiant power leaving the elemental surface area dA propagates to the satellite, and (neglecting any atmospheric absorption or scattering) enters the sensor aperturearea A. Hence the area dA at the Earth's surface and the sensor aperture A conform to an elementary beam system.

(insert figure 2.6 and caption here)

Figure 2.6. *Schematic of a space-based radiometer of aperture A and detector of area D viewing the ocean surface with the solid angles* Ω_s *and* Ω_d *referencing the sea and detector respectively.*

Thus the radiant power appears to originate at the aperture. Since,

$$W_{\mathbf{D}} = \mathbf{D / d^2},$$

the radiance passing through the aperture toward the detector may be written,

$$\mathbf{L = d}f \ \mathbf{/ A}W_{\mathbf{D}} \ = \ \mathbf{d}f \ \mathbf{/ (A.D/d2)\ .}$$

However, A / d^2 is the solid angle Ω_S at the detector, and

$$\mathbf{L = d}f \ \mathbf{/ (D.}\, W_{\mathbf{S}}\, \mathbf{).}$$

This is a key result in that it shows that if we make a measurement of the radiance based on thegeometry (design) of the sensor, the elementary beam concept indicates that this radiance is identical to that of the target we are observing on the Earth's surface!

4.0 Vector and Scalar Quantities

Later in the Workshop terms like scalar or vector irradiance will be introduced as will conversion from radiance to irradiance and vice versa. The important quantity normalised water leaving radiance will be encountered frequently in other modules and the definitions of these new quantities from the basic variables will be developed. On these more detailed issues, the text by Kirk (1986) is a particularly useful resource.

6.0 Concluding Remarks

For thisModule, the aim has been to introduce the basic concepts and to account for how and why these quantities are defined as they are. . While some of the concepts, such as the solid angle, may be new, particularly with respect to its use in measurment, it is clear that solid angles are more commonly encountered in our three dimensional world of observations than two-dimensional angles. The importance of the elementary beam concept to radiometry, and in particular to space-based radiometry, is hopefully clear. It ought also be clear as to why the basic quantity - radiance - is almost the natural variable for use in making Earth observations with instruments. When we encounter radiometry applied to remote sensing of the oceans, issues such as optical propagation across the air-water interface and the processes of scattering and absorption in the ocean water itself as well as particulate material and pigments complicates the science. Nevertheless, at all stages, the processes involved and their impact on a propagating beam need to be represented in terms of signal amplitudes, propagation directions and just a few basic quantities and their associated units.

7.0 References

Kirk, J. T. O. 1986. *Light and Photosynthesis in Aquatic Ecosystems*.(Cambridge University Press).

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